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Massive IIA supergravities

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Abstract: We perform a systematic search for all possible massive deformations of IIA supergravity in ten dimensions. We show that there exist exactly two possibilities: Romans supergravity and Howe-Lambert-West supergravity. Along the way we give the full details of the ten-dimensional superspace formulation of the latter. The scalar superfield at canonical mass dimension zero (whose lowest component is the dilaton), present in both Romans and massless IIA supergravities, is not introduced from the outset but its existence follows from a certain integrability condition implied by the Bianchi identities. This fact leads to the possibility for a certain topological modification of massless IIA, reflecting an analogous situation in eleven dimensions.

Keywords: M-Theory, Superspaces, Supergravity Models.

Contents

1. Introduction

Romans massive supergravity [1] has attracted a lot of interest following the observation that its mass parameter (cosmological constant) may be thought of as sourced by the D8 brane of type IIA [2]. If string theory is to be understood as embedded in M-theory, it would be desirable to have an eleven-dimensional understanding of Romans supergravity. The latter, however, has no covariant eleven-dimensional lift,[1] owing to certain no-go theorems forbidding any straightforward introduction of mass in eleven-dimensions [6].

There does exist however a topological modification of eleven-dimensional supergravity (subsequently dubbed 'MM-theory'), as pointed out by Howe in [7], which allows the introduction of a mass parameter upon compactification to ten dimensions. In this way one obtains the Howe-Lambert-West supergravity of reference [8]. The latter contains a

[1]See [3] for a noncovariant embedding of Romans supergravity in eleven-dimensions, and [4] for a recent implementation of the same idea in eleven-dimensional superspace. In [5] it was argued that although Romans massive IIA supergravity cannot be embedded in ordinary eleven-dimensional supergravity, massive IIA *string theory* can be embedded in M-theory.

lot of interesting physics [9 – 11]; it is nevertheless much less studied despite that, contrary to Romans supergravity, it has a well-understood covariant eleven-dimensional origin and it has been shown to admit a de Sitter vacuum.[2] HLW supergravity can also be obtained by a generalized Scherk-Schwarz reduction of the equations-of-motion of ordinary eleven-dimensional supergravity [12].

It would be desirable to have an understanding of the relation between Romans and HLW supergravity from a purely ten-dimensional perspective. We would also like to know how unique these supergravities are and whether there exist or not other massive deformations of type IIA.

In this paper we shall address these questions by working in ten-dimensional IIA superspace. The starting point of our search is the supertorsion Bianchi identities (BI) — which every supersymmetric system should satisfy. In this purely-geometric approach no form superfields are introduced by hand, unlike in the usual superspace formulation of IIA supergravity [13]; the field-strengths of the various supergravity forms 'sit' inside the components of the torsion. We do not make any further assumptions other than that any deviation from massless IIA should appear at canonical mass dimension one or higher. I.e. we demand that up to dimension one-half, the supertorsion components are (equivalent to) those of massless IIA supergravity. As there are formulations of the latter (see for example [14]) in which the scalar superfield does *not* appear explicitly in the torsion components of dimension zero or one-half, we shall assume that the field content at dimension zero consists of *at most* one scalar superfield, while the field content at dimension one-half consists of one chiral and one antichiral Majorana spinor superfields.

In fact, an interesting feature of the formulation presented here is that the scalar superfield at dimension zero (whose lowest component can be identified with the dilaton), present in both Romans and massless IIA supergravities, is *not* introduced from the outset but it arises as a 'potential' for the spinor superfields at dimension one-half: its existence follows from a certain integrability condition implied by the BI's. In the case of HLW supergravity the aforementioned integrability condition fails and such a 'potential' does not exist. More specifically: as we show at the end of section 3.4, both in the case of massless IIA and in Romans supergravity, one can construct a *closed* one-form superfield whose lowest component is identified with the spinor superfields at dimension one-half. In a topologically nontrivial spacetime this one-form may not be exact, a fact which leads to a topological modification of massless IIA. This is the ten-dimensional version of the possibility to modify ordinary eleven-dimensional supergravity to MM-theory.

We generally expect that taking the supertorsion Bianchi identities as the starting point, should allow for more freedom than starting with superforms. Evidence for this was recently provided in [15], where the supertorsion BI's of eleven-dimensional supergravity where solved at first order in a deformation parameter related to the Planck length.[3]

[2] Presumably this apparent neglect should be attributed to certain unconventional features of HLW supergravity. For example, its equations-of-motion cannot be integrated to a local lagrangian. More importantly, it is not clear at present if the theory can be made quantum-mechanically self-consistent.

[3] The first supersymmetric deformation occurs at order l^3 and it is of topological nature [16]. The next deformation, related to the R^4 superinvariant, is expected to occur at order l^6.

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Although it has been shown that the four-form formulation of 11d supergravity implies the supertorsion formulation [17],[4] it was realized in [15] that the converse may not be true.

The main result of this paper can be stated as follows. Depending on the values of two scalar superfields (L, L' of equation (4.7) below) arising at canonical mass dimension one, there exist exactly two massive deformations of IIA supergravity: Romans supergravity and HLW supergravity.

In the following section we introduce IIA superspace and establish our notation and conventions. We also examine the possible field redefinitions, in preparation for the analysis of the BI's in section 3. The reader who is not interested in the derivation of the final result, may skip directly to section 4 where the outcome of the analysis of the BI's is summarized. We conclude in section 5 with some possible future directions. The appendix contains our conventions on certain gamma-traceless projections used in section 3.

2. General setup

2.1 Type IIA superspace

Let us begin by introducing the usual superspace machinery of vielbein (E^A), connection ($\Omega_A{}^B$), torsion (T^A) and curvature ($R_A{}^B$), via

$$\begin{aligned} T^A &= DE^A \\ R_A{}^B &= d\Omega_A{}^B + \Omega_A{}^C \wedge \Omega_C{}^B \,. \end{aligned} \tag{2.1}$$

A flat superindex is denoted by a capital Latin letter from the beginning of the alphabet and stands for both bosonic (a) and fermionic ($\underline{\alpha}$) indices. Underlined Greek indices from the beginning of the alphabet stand for flat fermionic indices of both chiralities. For example:

$$S^{\underline{\alpha}} := (S^\alpha, S_\alpha)\,, \tag{2.2}$$

where S^α is chiral and S_α is antichiral. Note that we never raise/lower chiral fermionic indices, so that the position of the index denotes a definite chirality. In IIA superspace the spinor part of the vielbein contains both chiralities: $E^{\underline{\alpha}} := (E^\alpha, E_\alpha)$.

The torsion and curvature satisfy the Bianchi identities

$$\begin{aligned} DT^A &= E^B R_B{}^A \\ DR_A{}^B &= 0\,. \end{aligned} \tag{2.3}$$

If the connection is lorentzian, as we assume to be the case in the present paper, the second BI follows from the first [20]. Hence we need only analyze the first of equations (2.3), i.e. the supertorsion BI.

[4] This was shown in [17] to first order in the deformation parameter. In the undeformed theory, this fact had been previously pointed out in [18, 19].

2.2 Field redefinitions

Before coming to the analysis of the BI's, let us examine what the possible field redefinitions are. We have at our disposal vielbein and connection redefinitions which can be used to gauge-fix some of the torsion components. Specifically, let $h_A{}^B := E_A{}^M \delta E_M{}^B$ and $\Delta_{AB}{}^C := E_A{}^M \delta\Omega_{MB}{}^C$. Under $E_M{}^A \to E_M{}^A + \delta E_M{}^A$, $\Omega_{MB}{}^C \to \Omega_{MB}{}^C + \delta\Omega_{MB}{}^C$ the torsion transforms as $T_{AB}{}^C \to T_{AB}{}^C + \delta T_{AB}{}^C$ where

$$\delta T_{AB}{}^C = 2\Delta_{[AB\}}{}^C + 2D_{[A}h_{B\}}{}^C - 2h_{[A|}{}^D T_{D|B\}}{}^C + T_{AB}{}^D h_D{}^C \,. \tag{2.4}$$

Let us analyze the possible field redefinitions in the order of increasing canonical mass dimension.

Dimension 0. We search for supergravities which, in the massless limit, reduce to massless type IIA. Therefore, as explained in the introduction, we shall assume that the field content at dimension zero consists of *at most* a scalar superfield (ϕ). Hence, the most general form of the dimension-zero torsion components is

$$\begin{aligned} T_{\alpha\beta}{}^c &= c_1(\gamma^c)_{\alpha\beta} \\ T^{\alpha\beta c} &= c_2(\gamma^c)^{\alpha\beta} \\ T_\alpha{}^{\beta c} &= 0 \,. \end{aligned} \tag{2.5}$$

If a dimension-zero scalar superfield ϕ exists, c_1, c_2 can be arbitrary functions of ϕ. From (2.4) we see that we can use two independent linear combinations of $h_\alpha{}^\beta$, $h^\alpha{}_\beta$, $h_a{}^b$ to set $T_{\alpha\beta}{}^c = -i(\gamma^c)_{\alpha\beta}$, $T^{\alpha\beta c} = -i(\gamma^c)^{\alpha\beta}$. Note that in the case where there exists no scalar superfield ϕ at dimension zero, the remaining linear h-combination is a constant. Therefore it *cannot* be used as a redefinition at dimension one-half or higher, as Dh vanishes.

Dimension $\frac{1}{2}$**.** At canonical mass dimension one-half the field content consists of one right-handed and one left-handed Majorana spinor superfield μ^α, λ_α, respectively.[5] In the massless IIA limit we have $\lambda_\alpha = D_\alpha\phi$, $\mu^\alpha = D^\alpha\phi$. As we have already remarked, we shall not assume this to be the case *a priori*. The most general form of the vielbein and connection redefinitions is

$$\begin{aligned} \Delta_{\alpha b}{}^c &= d_1(\gamma_b{}^c\lambda)_\alpha \\ \Delta^\alpha{}_b{}^c &= d_2(\gamma_b{}^c\mu)^\alpha \end{aligned} \tag{2.6}$$

and

$$\begin{aligned} h_a{}^\alpha &= f_1(\gamma_a\lambda)^\alpha \\ h_{a\alpha} &= f_2(\gamma_a\mu)_\alpha \end{aligned} \tag{2.7}$$

[5]Here as well one can imagine the possibility that the dimension one-half spinors may be 'eaten' by a higher-dimensional superfield. However it is not difficult to see that in this case the Bianchi identities set all the torsion components to zero.

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respectively. Note that $\Delta_{\underline{\alpha}\beta}{}^{\gamma}$ is not independent, but is related to $\Delta_{\underline{\alpha}b}{}^{c}$ via the Lorentz condition. The most general form of the torsion component $T_{\alpha b}{}^{c}$ reads

$$T_{\alpha b}{}^{c} = e_1 \delta_b{}^c \lambda_\alpha + e_2 (\gamma_b{}^c \lambda)_\alpha \tag{2.8}$$

and similarly for $T^{\alpha}{}_{b}{}^{c}$. Hence we can use the field redefinitions (2.6), (2.7) to set $T_{\alpha b}{}^{c} = T^{\alpha}{}_{b}{}^{c} = 0$.

In the case where there exists no scalar superfield at zero dimension, this is all we can do in the way of gauge-fixing. However, when ϕ exists $T_{\alpha\beta}{}^{\gamma}$ can also be partially gauge-fixed as follows. The most general form of $T_{\alpha\beta}{}^{\gamma}$ is

$$T_{\alpha\beta}{}^{\gamma} = g_1 \lambda_{(\alpha} \delta_{\beta)}{}^{\gamma} + g_2 (\gamma^e)_{\alpha\beta} (\gamma_e \lambda)^{\gamma} . \tag{2.9}$$

As we can see from (2.4) and the fact that $D_\alpha \phi = \lambda_\alpha$, the remaining independent linear combination of $h_\alpha{}^\beta$, $h^\alpha{}_\beta$, $h_a{}^b$ can be used to gauge-fix the coefficient $g_1(\phi)$ in (2.9).

Dimension 1. As usual, $\Delta_{ab}{}^{c}$ can be used to set $T_{ab}{}^{c} = 0$.

To summarize the results of this subsection: the field redefinitions can be used to set

$$\begin{aligned} T_{\alpha\beta}{}^{c} &= -i(\gamma^c)_{\alpha\beta} \\ T^{\alpha\beta c} &= -i(\gamma^c)^{\alpha\beta} \\ T_{\underline{\alpha} b}{}^{c},\ T_{ab}{}^{c} &= 0 . \end{aligned} \tag{2.10}$$

If, in addition, there is a scalar superfield ϕ such that $D_\alpha \phi = \lambda_\alpha$, then $T_{\alpha\beta}{}^{\gamma}$ can also be partially gauge-fixed, as explained below (2.9).

3. Analysis of the torsion Bianchi identities

We are now ready to come to the solution of the torsion Bianchi identities. We shall proceed systematically, in increasing order of canonical mass dimension. The readers who are not interested in the details of this analysis, may skip directly to section 4 where the final result is summarized.

3.1 Dimension-$\frac{1}{2}$ BI

Taking the discussion of section 2.2 into account and imposing the Lorentz condition, the dimension one-half torsion Bianchi identity reads

$$T_{(\underline{\alpha}\underline{\beta}|}{}^{\underline{\epsilon}} T_{\underline{\epsilon}|\underline{\gamma})}{}^{e} = 0 . \tag{3.1}$$

We distinguish the following cases.

- Case 1: $(\underline{\alpha}, \underline{\beta}, \underline{\gamma}) = (\alpha, \beta, \gamma)$.

  The Bianchi identity (3.1) reads

$$T_{(\alpha\beta|}{}^{\epsilon} (\gamma^e)_{\epsilon|\gamma)} = 0 . \tag{3.2}$$

Substituting (2.9) in the equation above and using the identity

$$\gamma^e_{(\alpha\beta}\delta^\delta_{\gamma)} = (\gamma_f)_{(\alpha\beta}(\gamma^{ef})_{\gamma)}{}^\delta\,, \tag{3.3}$$

we obtain

$$T_{\alpha\beta}{}^\gamma = g_1\left\{\lambda_{(\alpha}\delta_{\beta)}{}^\gamma - \frac{1}{2}(\gamma^e)_{\alpha\beta}(\gamma_e\lambda)^\gamma\right\}. \tag{3.4}$$

- Case 2: $(\underline{\alpha}, \underline{\beta}, \underline{\gamma}) = ({}^\alpha, {}^\beta, {}^\gamma)$.

Similarly to the previous case we obtain

$$T^{\alpha\beta}{}_\gamma = g_2\Big\{\mu^{(\alpha}\delta^{\beta)}{}_\gamma - \frac{1}{2}(\gamma_e)^{\alpha\beta}(\gamma^e\mu)_\gamma\Big\}. \tag{3.5}$$

- Case 3: $(\underline{\alpha}, \underline{\beta}, \underline{\gamma}) = (\alpha, \beta, {}^\gamma)$.

The Bianchi identity (3.1) reads

$$2T_{(\alpha|}{}^{\gamma\epsilon}(\gamma^e)_{\epsilon|\beta)} + T_{\alpha\beta\epsilon}(\gamma^e)^{\epsilon\gamma} = 0\,. \tag{3.6}$$

We expand the torsion components above as follows

$$\begin{aligned} T_{\alpha\beta\gamma} &= g_3(\gamma^e)_{\alpha\beta}(\gamma_e\mu)_\gamma \\ T_\alpha{}^{\beta\gamma} &= g_4\delta_\alpha{}^\gamma\mu^\beta + g_5(\gamma^{e_1e_2})_\alpha{}^\gamma(\gamma_{e_1e_2}\mu)^\beta + g_6(\gamma^{e_1\dots e_4})_\alpha{}^\gamma(\gamma_{e_1\dots e_4}\mu)^\beta\,. \end{aligned}$$

In terms of irreducible representations the Bianchi identity decomposes as[6]

$$(00001)^{2\otimes_s} \otimes (00010) \otimes (10000) \sim 3(00010) \oplus \cdots\,,$$

where the ellipses stand for irreducible representations which drop out of the BI. Hence the BI imposes at most three independent conditions on the coefficients $g_3, \dots,$ g_6. These conditions can be obtained by contracting with the three independent structures $(\gamma^e)^{\alpha\beta}\delta^\epsilon_\gamma$, $(\gamma_f)^{\alpha\beta}(\gamma^{ef})_\gamma{}^\epsilon$ and $(\gamma^{ef_1\dots f_4})^{\alpha\beta}(\gamma_{f_1\dots f_4})_\gamma{}^\epsilon$. We thus obtain $g_4 = -\frac{1}{2}g_3$, $g_5 = \frac{1}{4}g_3$ and $g_6 = 0$, so that

$$\begin{aligned} T_{\alpha\beta\gamma} &= g_3(\gamma^e)_{\alpha\beta}(\gamma_e\mu)_\gamma \\ T_\alpha{}^{\beta\gamma} &= -\frac{1}{2}g_3\Big\{\delta_\alpha{}^\gamma\mu^\beta - \frac{1}{2}(\gamma^{ef})_\alpha{}^\gamma(\gamma_{ef}\mu)^\beta\Big\}. \end{aligned} \tag{3.7}$$

- Case 4: $(\underline{\alpha}, \underline{\beta}, \underline{\gamma}) = (\alpha, {}^\beta, {}^\gamma)$.

Similarly to the previous case, we obtain

$$\begin{aligned} T^{\alpha\beta\gamma} &= g_4(\gamma_e)^{\alpha\beta}(\gamma^e\lambda)^\gamma \\ T^\alpha{}_{\beta\gamma} &= -\frac{1}{2}g_4\Big\{\delta^\alpha{}_\gamma\lambda_\beta - \frac{1}{2}(\gamma^{ef})^\alpha{}_\gamma(\gamma_{ef}\lambda)_\beta\Big\}. \end{aligned} \tag{3.8}$$

[6]We are using the Dynkin notation for the complex cover D_5 of SO(1, 9). I.e. (00000), (10000), (01000), (00100), (00011) denote a scalar, vector, two-form, three-form, four-form respectively. Self-dual, anti-self-dual five-forms are denoted by (00002), (00020) respectively and chiral, anti-chiral spinors are denoted by (00010), (00001). Similarly, an irreducible (gamma-traceless) chiral vector-spinor is denoted by (10010), a chiral two-form spinor by (01010), etc.

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3.2 Dimension-1 BI

For the gamma-matrix manipulations of this and the remaining subsections, we have found [21] extremely useful. We have also made use of [22] in evaluating tensor products of representations.

Let us expand the spinor derivatives of the dimension-$\frac{1}{2}$ spinor superfields as follows

$$\begin{aligned} D_\alpha \lambda_\beta &= K_e(\gamma^e)_{\alpha\beta} + K_{efg}(\gamma^{efg})_{\alpha\beta} \\ D_\alpha \mu^\beta &= L\delta_\alpha{}^\beta + L_{ef}(\gamma^{ef})_\alpha{}^\beta + L_{efgh}(\gamma^{efgh})_\alpha{}^\beta \end{aligned} \tag{3.9}$$

and

$$\begin{aligned} D^\alpha \lambda_\beta &= L'\delta^\alpha{}_\beta + L'^{ef}(\gamma_{ef})^\alpha{}_\beta + L'^{efgh}(\gamma_{efgh})^\alpha{}_\beta \\ D^\alpha \mu^\beta &= K'^e(\gamma_e)^{\alpha\beta} + K'^{efg}(\gamma_{efg})^{\alpha\beta} \,. \end{aligned} \tag{3.10}$$

We also expand the dimension-one torsion as follows:

$$\begin{aligned} T_{ab}{}^c &= 0 \\ T_{a\alpha}{}^\beta &= \delta_\alpha{}^\beta V^1_a + (\gamma_a{}^e)_\alpha{}^\beta V^2_e + (\gamma_a{}^{fgh})_\alpha{}^\beta H^1_{fgh} + (\gamma^{fg})_\alpha{}^\beta H^2_{afg} \\ T_a{}^\alpha{}_\beta &= \delta^\alpha{}_\beta V'^1_a + (\gamma_a{}^e)^\alpha{}_\beta V'^2_e + (\gamma_a{}^{fgh})^\alpha{}_\beta H'^1_{fgh} + (\gamma^{fg})^\alpha{}_\beta H'^2_{afg} \\ T_{a\alpha\beta} &= S(\gamma_a)_{\alpha\beta} + (\gamma_a{}^{ef})_{\alpha\beta} F^1_{ef} + (\gamma^e)_{\alpha\beta} F^2_{ae} + (\gamma_a{}^{efgh})_{\alpha\beta} G^1_{efgh} + (\gamma^{efg})_{\alpha\beta} G^2_{aefg} \\ T_a{}^{\alpha\beta} &= S'(\gamma_a)^{\alpha\beta} + (\gamma_a{}^{ef})^{\alpha\beta} F'^1_{ef} + (\gamma^e)^{\alpha\beta} F'^2_{ae} + (\gamma_a{}^{efgh})^{\alpha\beta} G'^1_{efgh} + (\gamma^{efg})^{\alpha\beta} G'^2_{aefg} \,. \end{aligned} \tag{3.11}$$

The superfields appearing on the right-hand-side of (3.11) are all forms. We can see that there can be no hooks in the above expansions, for the following reason. Assuming there is a hook superfield (U) at dimension one, we can expand $U = mU_{(0)} + U_{(1)}$, where m is a mass parameter, so that $U_{(0)}$ is of canonical mass dimension zero, $U_{(1)}$ is of canonical mass dimension one and does not depend on m. Taking the massless $m \to 0$ limit we see that $U_{(1)}$ has to vanish, as no hook superfields can appear in the torsion components of massless IIA. Also $U_{(0)}$ has to vanish, since at dimension zero there can exist at most a scalar superfield. Using the same argument we can see that there can be no five-forms in the expansions (3.9)–(3.11).

Taking the Lorentz condition into account, the Bianchi identities at dimension one read

$$R_{\underline{\alpha}\underline{\beta}cd} = 2T_{c(\underline{\alpha}|}{}^{\underline{\epsilon}}T_{\underline{\epsilon}|\underline{\beta})d} \,. \tag{3.12}$$

and

$$R_{(\underline{\alpha}\underline{\beta}\underline{\gamma})}{}^{\underline{\delta}} = D_{(\underline{\alpha}}T_{\underline{\beta}\underline{\gamma})}{}^{\underline{\delta}} + T_{(\underline{\alpha}\underline{\beta}|}{}^eT_{e|\underline{\gamma})}{}^{\underline{\delta}} + T_{(\underline{\alpha}\underline{\beta}|}{}^{\underline{\epsilon}}T_{\underline{\epsilon}|\underline{\gamma})}{}^{\underline{\delta}} \,. \tag{3.13}$$

Let us analyze (3.12) first. We distinguish the following cases.

- Case 1: $(\underline{\alpha}, \underline{\beta}) = (\alpha, \beta)$.

  Demanding that the right-hand-side of (3.12) be antisymmetric in c, d implies

  $$V^1_a = V^2_a = 0 \tag{3.14}$$

  and the curvature is given by

  $$R_{\alpha\beta cd} = 2i(\gamma_{cd}{}^{efg})_{\alpha\beta} H^1_{efg} + 4i(\gamma^e)_{\alpha\beta} H^2_{cde} \,. \tag{3.15}$$

- Case 2: $(\underline{\alpha},\underline{\beta}) = ({}^{\alpha},{}^{\beta})$.

  Similarly to the previous case we get

$$V'^1_a = V'^2_a = 0 \tag{3.16}$$

  and

$$R^{\alpha\beta}{}_{cd} = 2i(\gamma_{cd}{}^{efg})^{\alpha\beta}H'^1_{efg} + 4i(\gamma^e)^{\alpha\beta}H'^2_{cde}\,. \tag{3.17}$$

- Case 3: $(\underline{\alpha},\underline{\beta}) = (\alpha,{}^{\beta})$.

  Demanding that the right-hand-side of (3.12) be antisymmetric in c, d in this case implies

$$\begin{aligned} S &= 7 - S' \\ F^1 &= F'^1 \\ F^2 &= F'^2 \\ G^1 &= -G'^1 \\ G^2 &= -G'^2 \end{aligned} \tag{3.18}$$

  and the curvature is given by

$$R_{\alpha}{}^{\beta}{}_{cd} = -2i\Big\{(\gamma_{cd})_{\alpha}{}^{\beta}S + (\gamma_{cd}{}^{ef})_{\alpha}{}^{\beta}F^1_{ef} + \delta_{\alpha}{}^{\beta}F^2_{cd} + (\gamma_{cd}{}^{efgh})_{\alpha}{}^{\beta}G^1_{efgh} + 3(\gamma^{fg})_{\alpha}{}^{\beta}G^2_{cdfg}\Big\}\,. \tag{3.19}$$

  Let us now come to (3.13). We distinguish the following cases.

- Case 1.1: $(\underline{\alpha},\underline{\beta},\underline{\gamma},\underline{\delta}) = (\alpha,\beta,\gamma,\delta)$.

  We shall work out this case in some detail in order to illustrate the general procedure. Taking the Lorentz condition into account, we obtain

$$\begin{aligned} &(\gamma^e)_{(\alpha\beta}(\gamma_e{}^{fgh})_{\gamma)}{}^{\delta}\Big\{iH^1_{fgh} - \frac{1}{2}g_1K_{fgh} - \frac{1}{96}(g_1^2 + \frac{\dot g_1}{2})(\lambda\gamma_{fgh}\lambda)\Big\} + \\ &+(\gamma^e)_{(\alpha\beta}(\gamma^{fg})_{\gamma)}{}^{\delta}\Big\{2iH^2_{efg} + \frac{3}{2}g_1K_{efg} - \frac{1}{4}(\mu\gamma_{efg}\mu) - \frac{3}{96}(g_1^2 - \frac{\dot g_1}{2})(\lambda\gamma_{efg}\lambda)\Big\} + \\ &+\frac{i}{2}(\gamma_{cd}{}^{efg})_{(\alpha\beta}(\gamma^{cd})_{\gamma)}{}^{\delta}H^1_{efg} - \frac{1}{2}g_1K^f\Big\{(\gamma_f)_{(\alpha\beta}\delta^{\delta}_{\gamma)} + (\gamma^e)_{(\alpha\beta}(\gamma_{ef})_{\gamma)}{}^{\delta}\Big\} = 0\,, \end{aligned} \tag{3.20}$$

  where we have allowed for the possibility that $\lambda_\alpha = D_\alpha\phi$ and we have set $\dot g_1 := \frac{d}{d\phi}g_1$. Note that the vector drops out of the BI due to the identity

$$(\gamma_f)_{(\alpha\beta}\delta^{\delta}_{\gamma)} = -(\gamma^e)_{(\alpha\beta}(\gamma_{ef})_{\gamma)}{}^{\delta}\,.$$

  Moreover, one can see that there is a unique three-form in the decomposition of the tensor product of a chiral spinor and the symmetrized tensor product of three antichiral spinors:

$$(00010)\otimes(00001)^{3\otimes_s} \sim 1(10000)\oplus 1(00100)\oplus\cdots\,. \tag{3.21}$$

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Therefore, equation (3.20) imposes at most one linear equation on the three-forms. Contracting both sides of (3.20) with $(\gamma_{[a})^{\alpha\beta}(\gamma_{bc]})_{\delta}{}^{\gamma}$ in order to saturate the spinor indices, we obtain[7]

$$48iH^1_{abc} - 8iH^2_{abc} - 12g_1K_{abc} + (\mu\gamma_{abc}\mu) + \frac{1}{16}(g_1^2 - 2\dot{g}_1)(\lambda\gamma_{abc}\lambda) = 0\,. \tag{3.22}$$

- Case 1.2: $(\underline{\alpha},\underline{\beta},\underline{\gamma},\underline{\delta}) = (\alpha,\beta,\gamma,\overset{\delta}{\ })$.

  In terms of irreducible representations the BI decomposes as

$$(00001) \otimes (00001)^{3\otimes_s} \sim 1(01000) \oplus 1(00011) \oplus \cdots\,. \tag{3.23}$$

  Hence the BI imposes at most one linear equation on the two-forms and one on the four-forms. Contracting with $(\gamma_{[a})^{\alpha\beta}(\gamma_{b]})^{\delta\gamma}$ we obtain

$$0 = L_{ab} + \frac{1}{4}(\mu\gamma_{ab}\lambda) - \frac{i}{2}F^1_{ab} + \frac{i}{4}F^2_{ab}\,. \tag{3.24}$$

  Similarly, contracting with $(\gamma_{[a})^{\alpha\beta}(\gamma_{bcd]})^{\delta\gamma}$ we obtain

$$0 = L_{abcd} - \frac{i}{2}G^1_{abcd} + \frac{i}{8}G^2_{abcd}\,. \tag{3.25}$$

- Case 2.1: $(\underline{\alpha},\underline{\beta},\underline{\gamma},\underline{\delta}) = (\alpha,\beta,\overset{\gamma}{\ },\delta)$.

  In terms of representations, the BI decomposes as

$$(00010)^{2\otimes} \otimes (00001)^{2\otimes_s} \sim 2(00000) \oplus 4(01000) \oplus 5(00011) \oplus \cdots\,. \tag{3.26}$$

  I.e. the BI imposes at most two constraints on the scalars, four on the two-forms and five on the four-forms. Let us examine each representation in turn.

  Scalars: using the independent structures $(S_1)^{\alpha\beta}_{\gamma\delta} := (\gamma^a)^{\alpha\beta}(\gamma_a)_{\gamma\delta}$ and $(S_2)^{\alpha\beta}_{\gamma\delta} := (\gamma^{a_1\ldots a_5})^{\alpha\beta}(\gamma_{a_1\ldots a_5})_{\gamma\delta}$ to saturate the spinor indices,[8] we obtain

$$S = -\frac{2i}{5}L - \frac{ig_1}{10}L' + i\Big(\frac{1}{20} - \frac{11g_1}{80} - \frac{\dot{g}_1}{160}\Big)(\mu\lambda)\,. \tag{3.27}$$

  Hence in this case both independent contractions yield the same equation.

[7] One can verify that the same equation is obtained by contracting (3.20) with, for example, $(\gamma^e)^{\alpha\beta}(\gamma_{eabc})_{\delta}{}^{\gamma}$. Here and in the rest of the analysis of the BI's, we have applied many more contractions than the number of independent ones. This is not strictly-speaking necessary, but the 'redundant ' contractions serve as useful consistency checks.

[8] The fact that these structures are independent is seen as follows. Contracting the equation

$$A(\gamma^a)_{\alpha\beta}(\gamma_a)^{\gamma\delta} + B(\gamma^{a_1\cdots a_5})_{\alpha\beta}(\gamma_{a_1\ldots a_5})^{\gamma\delta} = 0$$

with S_1, S_2, we obtain $A = B = 0$. Note that there are exactly two independent structures as follows from the representation-theoretic analysis (3.26).

Two-forms: contracting with the four independent structures $(\gamma_{[a})^{\alpha\beta}\ (\gamma_{b]})_{\gamma\delta}$, $(\gamma^{f})^{\alpha\beta}\ (\gamma_{fab})_{\gamma\delta}$, $(\gamma^{abfgh})^{\alpha\beta}\ (\gamma_{fgh})_{\gamma\delta}$ and $(\gamma^{[a|efgh})^{\alpha\beta}(\gamma^{|b]}{}_{efgh})_{\gamma\delta}$, we get

$$
\begin{aligned}
0 &= \frac{g_1}{6}L'_{ab} + iF^1_{ab} + \Big(\frac{1}{8} + \frac{g_1}{32} + \frac{\dot{g}_1}{192}\Big)(\mu\gamma_{ab}\lambda) \\
0 &= 4L_{ab} + \frac{g_1}{3}L'_{ab} + iF^2_{ab} + \Big(\frac{5}{4} + \frac{g_1}{16} + \frac{\dot{g}_1}{96}\Big)(\mu\gamma_{ab}\lambda)\,.
\end{aligned}
\tag{3.28}
$$

Remarkably, all four independent contractions yield only the two constraints above. We also remark that equations (3.28) imply (3.24), which is therefore not independent.

Four-forms: contracting with the five independent structures $(\gamma^{[a})^{\alpha\beta}(\gamma^{bcd]})_{\gamma\delta}$, $(\gamma^{e})^{\alpha\beta}(\gamma_{e}{}^{abcd})_{\gamma\delta}$, $(\gamma^{abcde})^{\alpha\beta}(\gamma_{e})_{\gamma\delta}$, $(\gamma^{[abc|ef})^{\alpha\beta}(\gamma^{|d]}{}_{ef})_{\gamma\delta}$ and $(\gamma^{[ab|efg})^{\alpha\beta}(\gamma^{cd]}{}_{efg})_{\gamma\delta}$ we get

$$
\begin{aligned}
0 &= 2L_{abcd} - \frac{g_1}{2}L'_{abcd} + iG^1_{abcd} + \Big(\frac{1}{96} + \frac{g_1}{384} - \frac{\dot{g}_1}{768}\Big)(\mu\gamma_{abcd}\lambda) \\
0 &= 16L_{abcd} - 2g_1L'_{abcd} + iG^2_{abcd} + \Big(\frac{1}{24} + \frac{g_1}{96} - \frac{\dot{g}_1}{192}\Big)(\mu\gamma_{abcd}\lambda)\,.
\end{aligned}
\tag{3.29}
$$

I.e. all five independent contractions yield only the two constraints above. Moreover equations (3.29) imply (3.25).

- Case 2.2: $(\underline{\alpha}, \underline{\beta}, \underline{\gamma}, \underline{\delta}) = (\alpha, \beta, {}^{\gamma}, {}^{\delta})$.

The BI decomposes as

$$
(00001)^{2\otimes_s} \otimes (00001) \otimes (00010) \sim 3(10000) \oplus 4(00100) \oplus \cdots\,.
\tag{3.30}
$$

Therefore the BI will yield at most three constraints on the vectors and four on the three-forms. It can be seen that the vector part of the BI is equivalent to

$$
K_a = K'_a\,.
\tag{3.31}
$$

Moreover, contracting with the four independent structures $(\gamma^{ef})_{\gamma}{}^{\delta}\,(\gamma_{ef}{}^{abc})^{\alpha\beta}$, $(\gamma^{[a|efg})_{\gamma}{}^{\delta}(\gamma_{efg}{}^{|bc]})^{\alpha\beta}$, $(\gamma^{[ab})_{\gamma}{}^{\delta}(\gamma^{c]})^{\alpha\beta}$ and $(\gamma^{abce})_{\gamma}{}^{\delta}(\gamma_{e})^{\alpha\beta}$ we obtain

$$
\begin{aligned}
0 &= iH^1_{abc} + K_{abc} + \Big(\frac{1}{48} + \frac{g_1}{192}\Big)(\lambda\gamma_{abc}\lambda) \\
0 &= iH^2_{abc} + iH'^2_{abc} - 3K_{abc} - 3K'_{abc} + \Big(\frac{3}{16} + \frac{7g_1}{64}\Big)(\mu\gamma_{abc}\mu) + \Big(\frac{3}{16} + \frac{7g_2}{64}\Big)(\lambda\gamma_{abc}\lambda) \\
0 &= iH'^1_{abc} + K'_{abc} + \Big(\frac{1}{48} + \frac{g_2}{192}\Big)(\mu\gamma_{abc}\mu)\,.
\end{aligned}
\tag{3.32}
$$

- Case 3.1: $(\underline{\alpha}, \underline{\beta}, \underline{\gamma}, \underline{\delta}) = (\alpha, {}^{\beta}, {}^{\gamma}, \delta)$.

In terms of representations, this is the same as case 2.2 above. The equations which follow from this BI turn out to be identical to (3.31), (3.32).

- Case 3.2: $(\underline{\alpha},\underline{\beta},\underline{\gamma},\underline{\delta}) = (\alpha, {}^{\beta}, {}^{\gamma}, {}^{\delta})$.

  In terms of representations, this is the same as case 2.1. Proceeding similarly, we obtain the following equations

  Scalars:

$$S = \frac{2i}{5}L' + \frac{ig_2}{10}L + i\Big(\frac{1}{20} - \frac{11g_2}{80} - \frac{\dot{g}_2}{160}\Big)(\mu\lambda)\,. \tag{3.33}$$

  Two-forms:

$$\begin{aligned} 0 &= \frac{g_2}{6}L_{ab} + iF^1_{ab} + \Big(\frac{1}{8} + \frac{g_2}{32} + \frac{\dot{g}_2}{192}\Big)(\mu\gamma_{ab}\lambda) \\ 0 &= 4L'_{ab} + \frac{g_2}{3}L_{ab} + iF^2_{ab} + \Big(\frac{5}{4} + \frac{g_2}{16} + \frac{\dot{g}_2}{96}\Big)(\mu\gamma_{ab}\lambda)\,. \end{aligned} \tag{3.34}$$

  Four-forms:

$$\begin{aligned} 0 &= 2L'_{abcd} - \frac{g_2}{2}L_{abcd} - iG^1_{abcd} - \Big(\frac{1}{96} + \frac{g_2}{384} + \frac{\dot{g}_1}{768}\Big)(\mu\gamma_{abcd}\lambda) \\ 0 &= 16L'_{abcd} - 2g_2L_{abcd} - iG^2_{abcd} - \Big(\frac{1}{24} + \frac{g_2}{96} + \frac{\dot{g}_2}{192}\Big)(\mu\gamma_{abcd}\lambda)\,. \end{aligned} \tag{3.35}$$

- Case 4.1: $(\underline{\alpha},\underline{\beta},\underline{\gamma},\underline{\delta}) = ({}^{\alpha}, {}^{\beta}, {}^{\gamma}, \delta)$.

  In terms of representations, this is the same as case 1.2. Proceeding similarly we obtain the following equations

$$0 = L'_{ab} + \frac{1}{4}(\mu\gamma_{ab}\lambda) - \frac{i}{2}F^1_{ab} + \frac{i}{4}F^2_{ab} \tag{3.36}$$

  and

$$0 = L'_{abcd} + \frac{i}{2}G^1_{abcd} - \frac{i}{8}G^2_{abcd}\,. \tag{3.37}$$

- Case 4.2: $(\underline{\alpha},\underline{\beta},\underline{\gamma},\underline{\delta}) = ({}^{\alpha}, {}^{\beta}, {}^{\gamma}, {}^{\delta})$.

  In terms of representations, this is the same as case 1.1. Proceeding similarly we obtain

$$48iH'^1_{abc} - 8iH'^2_{abc} - 12g_2K'_{abc} + (\lambda\gamma_{abc}\lambda) + \frac{1}{16}(g_2^2 - 2\dot{g}_2)(\mu\gamma_{abc}\mu) = 0\,. \tag{3.38}$$

  The system of equations above imply in particular

$$\begin{aligned} L_{ab} &= L'_{ab} \\ L_{abcd} &= -L'_{abcd} \end{aligned} \tag{3.39}$$

  and

$$\begin{aligned} g_1 - g_2 &= 0 \\ (g_1 + 4)(L + L') &= 0\,. \end{aligned} \tag{3.40}$$

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Conditions (3.31), (3.39) if supplemented with $L = -L'$, would imply $D_\alpha \mu^\beta = -D^\beta \lambda_\alpha$, $(\gamma_a)^{\alpha\beta} D_\alpha \lambda_\beta = (\gamma_a)_{\alpha\beta} D^\alpha \mu^\beta$. The latter two equations are solved for $\lambda_\alpha = D_\alpha \phi$, $\mu^\alpha = D^\alpha \phi$. We shall see in the following that in the case $g_1 = -4$, $L = -L'$, the existence of ϕ is indeed implied by the higher-dimension BI's. But we have seen in section 2.2 that in this case the coefficient g_1 can be shifted by field redefinitions. Hence the case $g_1 = -4$, $L = -L'$ is in fact equivalent to the case $g_1 \neq -4$, $L = -L'$. In the following we shall set $g_1 = -4$, so that we can treat both $L = -L'$ and $L \neq -L'$ cases simultaneously. Note that if $L \neq -L'$, there cannot exist a ϕ such that $\lambda_\alpha = D_\alpha \phi$, $\mu^\alpha = D^\alpha \phi$.

3.3 Dimension-$\frac{3}{2}$ BI

Taking into account our gauge-fixing in section 2.2, the Bianchi identities at canonical mass dimension three-half read

$$2R_{\underline{\alpha}[bc]}{}^d = T_{bc}{}^{\underline{\epsilon}} T_{\underline{\epsilon}\underline{\alpha}}{}^d \tag{3.41}$$

and

$$2R_{e(\underline{\alpha}\underline{\beta})}{}^{\underline{\delta}} = 2D_{(\underline{\alpha}} T_{\underline{\beta})e}{}^{\underline{\delta}} + D_e T_{\underline{\alpha}\underline{\beta}}{}^{\underline{\delta}} + T_{\underline{\alpha}\underline{\beta}}{}^f T_{fe}{}^{\underline{\delta}} + T_{\underline{\alpha}\underline{\beta}}{}^{\underline{\epsilon}} T_{\underline{\epsilon}e}{}^{\underline{\delta}} + 2T_{e(\underline{\alpha}}{}^{\underline{\epsilon}} T_{\underline{\epsilon}|\underline{\beta})}{}^{\underline{\delta}} . \tag{3.42}$$

Equation (3.41) can be solved for the dimension three-half supercurvature to give

$$R_{\underline{\alpha}bcd} = \frac{i}{2}\Big\{ (\gamma_b T_{cd})_{\underline{\alpha}} + (\gamma_c T_{bd})_{\underline{\alpha}} - (\gamma_d T_{bc})_{\underline{\alpha}} \Big\} . \tag{3.43}$$

In the following we shall expand the dimension-$\frac{3}{2}$ torsion into irreducible (gamma-traceless) parts as follows

$$T_{ab} = \widetilde{T}_{ab} + \gamma_{[a} \widetilde{T}_{b]} + \gamma_{ab} \widetilde{T} , \tag{3.44}$$

where we have suppressed all spinor indices. Similarly, we expand the spinor derivatives of the various dimension-one fields in irreducible representations, as follows (again suppressing spinor indices)

$$\begin{aligned}
DL &= \widetilde{L} \\
DL' &= \widetilde{L}' \\
DK_a &= \widetilde{K}^{(1)}_a + \gamma_a \widetilde{K}^{(1)} \\
DL_{ab} &= \widetilde{L}^{(2)}_{ab} + \gamma_{[a} \widetilde{L}^{(2)}_{b]} + \gamma_{ab} \widetilde{L}^{(2)} \\
DK_{abc} &= \widetilde{K}^{(3)}_{abc} + \gamma_{[a} \widetilde{K}^{(3)}_{bc]} + \gamma_{[ab} \widetilde{K}^{(3)}_{c]} + \gamma_{abc} \widetilde{K}^{(3)} \\
DL_{abcd} &= \widetilde{L}^{(4)}_{abcd} + \gamma_{[a} \widetilde{L}^{(4)}_{bcd]} + \gamma_{[ab} \widetilde{L}^{(4)}_{cd]} + \gamma_{[abc} \widetilde{L}^{(4)}_{d]} + \gamma_{abcd} \widetilde{L}^{(4)} .
\end{aligned} \tag{3.45}$$

Let us now come to equation (3.42). We distinguish the following cases.

- Case 1.1: $(\underline{\alpha}, \underline{\beta}, \underline{\delta}) = (\alpha, \beta, \delta)$

  In terms of irreducible representations the BI decomposes as

$$\begin{aligned}
&(10000) \otimes (00010) \otimes (00001)^{2\otimes_s} \sim \\
&\sim 3(00010) \oplus 4(10001) \oplus 2(01010) \oplus 2(00101) \oplus (00021) \oplus \cdots .
\end{aligned}$$

  Hence the BI imposes at most three constraints on the spinors, four on the vector-spinors, etc. Let us analyze each representation in turn

Spinors. Contracting with the three independent structures $(\gamma^e)^{\alpha\beta}\delta_\delta{}^\gamma$, $(\gamma_g)^{\alpha\beta}$ $(\gamma^{ge})_\delta{}^\gamma$ and $(\gamma^{eg_1\ldots g_4})^{\alpha\beta}(\gamma_{g_1\ldots g_4})_\delta{}^\gamma$, we obtain

$$\begin{aligned}
\widetilde{T} &= -\frac{14i}{135}\not{D}\lambda + \frac{32}{45}L\mu + \frac{8}{45}L'\mu + \frac{136}{405}L_{(2)}(\gamma^{(2)}\mu) + \frac{8}{45}L_{(4)}(\gamma^{(4)}\mu) - \\
&\quad - \frac{16}{45}K_{(1)}(\gamma^{(1)}\lambda) - \frac{2}{45}K_{(3)}(\gamma^{(3)}\lambda) - \frac{1}{60}(\mu\gamma_{(3)}\mu)(\gamma^{(3)}\lambda) \\
\widetilde{K}^{(3)} &= -\frac{i}{540}\not{D}\lambda + \frac{1}{180}L\mu + \frac{1}{45}L'\mu - \frac{13}{1620}L_{(2)}(\gamma^{(2)}\mu) + \frac{1}{180}L_{(4)}(\gamma^{(4)}\mu) - \\
&\quad - \frac{1}{90}K_{(1)}(\gamma^{(1)}\lambda) + \frac{1}{45}K_{(3)}(\gamma^{(3)}\lambda)\,,
\end{aligned} \tag{3.46}$$

where, to simplify the expressions, we have introduced the notation $\gamma_{(p)}A^{(p)} := \gamma_{a_1\ldots a_p}A^{a_1\ldots a_p}$.

Vector-spinors. Contracting the BI with the four independent structures $\delta_a{}^e(\gamma_g)^{\alpha\beta}$ $(\gamma^g)_{\delta\gamma}$, $\delta_a{}^e(\gamma_{g_1\ldots g_5})^{\alpha\beta}(\gamma^{g_1\ldots g_5})_{\delta\gamma}$, $(\gamma_a)^{\alpha\beta}(\gamma^e)_{\delta\gamma}$, $(\gamma_{ag_1\ldots g_4})^{\alpha\beta}(\gamma^{eg_1\ldots g_4})_{\delta\gamma}$ and projecting onto the gamma-traceless part, we obtain

$$\begin{aligned}
\widetilde{T}_a &= -\frac{3i}{20}(\gamma_a^{(1)}D_{(1)}\lambda) - \frac{1}{5}L_{(2)}(\gamma_a^{(2)}\mu) - \frac{2}{5}L_{(4)}(\gamma_a^{(4)}\mu) + \\
&\quad + \frac{1}{5}K_{(1)}(\gamma_a^{(1)}\lambda) + \frac{3}{20}K_{(3)}(\gamma_a^{(3)}\lambda) + \frac{3}{160}(\mu\gamma_{(3)}\mu)(\gamma_a^{(3)}\lambda) \\
\widetilde{K}_a^{(3)} &= \frac{i}{160}(\gamma_a^{(1)}D_{(1)}\lambda) + \frac{1}{40}L_{(2)}(\gamma_a^{(2)}\mu) + \frac{1}{20}L_{(4)}(\gamma_a^{(4)}\mu) - \\
&\quad - \frac{1}{40}K_{(1)}(\gamma_a^{(1)}\lambda) + \frac{81}{1120}K_{(3)}(\gamma_a^{(3)}\lambda) - \frac{1}{1280}(\mu\gamma_{(3)}\mu)(\gamma_a^{(3)}\lambda)\,.
\end{aligned} \tag{3.47}$$

The notation in the equation above is a shorthand for the projection onto the irreducible (gamma-traceless) vector-spinor part. Our conventions are explained in appendix A.

Two-form-spinors. Contracting with the two independent structures $(\gamma_{[a})^{\alpha\beta}$ $(\gamma_{b]}{}^e)_\delta{}^\gamma$, $(\gamma_{ab}{}^{eg_1g_2})^{\alpha\beta}(\gamma_{g_1g_2})_\delta{}^\gamma$, we obtain

$$\widetilde{K}_{ab}^{(3)} = \frac{1}{8}\widetilde{T}_{ab} + \frac{1}{108}L_{(2)}(\gamma_{ab}^{(2)}\mu) + \frac{1}{6}L_{(4)}(\gamma_{ab}^{(4)}\mu) + \frac{1}{18}K_{(3)}(\gamma_{ab}^{(3)}\lambda)\,. \tag{3.48}$$

As in the previous case, the projections onto the gamma-traceless part of the two-form-spinors are explained in appendix A.

Three-form-spinors. Contracting with the two independent structures $\delta_{[a}{}^e$ $(\gamma_{bc]g_1g_2g_3})^{\alpha\beta}(\gamma^{g_1g_2g_3})_{\delta\gamma}$ and $(\gamma_{abc}{}^{eg})^{\alpha\beta}(\gamma_g)_{\delta\gamma}$, we obtain

$$\widetilde{K}_{abc}^{(3)} = -\frac{1}{84}K_{(3)}(\gamma_{abc}^{(3)}\lambda)\,. \tag{3.49}$$

Four-form-spinors. Contracting with $(\gamma_{abcd}{}^e)^{\alpha\beta}\delta_\delta{}^\gamma$, we obtain

$$\widetilde{L}^{(4)}_{abcd} = 0\,. \tag{3.50}$$

- Case 1.2: $(\underline{\alpha}, \underline{\beta}, \underline{\delta}) = (\alpha, \beta, \overset{\delta}{})$.

In terms of irreducible representations the BI decomposes as

$$\begin{aligned}&(10000) \otimes (00001) \otimes (00001)^{2\otimes_s} \sim \\ &\quad \sim 3(00001) \oplus 3(10010) \oplus 3(01001) \oplus (00110) \oplus 2(00012) \oplus \cdots\end{aligned}$$

hence there will be at most three constraints on the spinors, three on the vector-spinors, etc. Let us examine each irreducible representation in turn:

Spinors. Contracting with the independent structures $(\gamma^e)^{\alpha\beta}\delta_\gamma{}^\delta$, $(\gamma_g)^{\alpha\beta}(\gamma^{ge})_\gamma{}^\delta$ and $(\gamma^{eg_1\ldots g_4})^{\alpha\beta}(\gamma_{g_1\ldots g_4})_\gamma{}^\delta$, we obtain

$$\begin{aligned}
\widetilde{L} &= \widetilde{L}' - \frac{i}{16}\not{D}\mu + \frac{25}{8}L\lambda + \frac{1}{4}L'\lambda + \frac{73}{24}L_{(2)}(\gamma^{(2)}\lambda) + \frac{9}{8}L_{(4)}(\gamma^{(4)}\lambda) - \\
&\quad - \frac{3}{8}K_{(1)}(\gamma^{(1)}\mu) - \frac{15}{16}K_{(3)}(\gamma^{(3)}\mu) - \frac{1}{64}(\lambda\gamma_{(3)}\lambda)(\gamma^{(3)}\mu) - \frac{135}{64}\widetilde{T} \\
\widetilde{L}^{(2)} &= -\frac{3i}{320}\not{D}\mu - \frac{21}{160}L\lambda + \frac{3}{80}L'\lambda + \frac{59}{480}L_{(2)}(\gamma^{(2)}\lambda) - \frac{1}{32}L_{(4)}(\gamma^{(4)}\lambda) - \\
&\quad - \frac{9}{160}K_{(1)}(\gamma^{(1)}\mu) + \frac{19}{320}K_{(3)}(\gamma^{(3)}\mu) - \frac{3}{1280}(\lambda\gamma_{(3)}\lambda)(\gamma^{(3)}\mu) - \frac{33}{256}\widetilde{T} \\
\widetilde{L}^{(4)} &= -\frac{i}{3840}\not{D}\mu - \frac{7}{1920}L\lambda + \frac{1}{960}L'\lambda - \frac{1}{3456}L_{(2)}(\gamma^{(2)}\lambda) - \frac{131}{40320}L_{(4)}(\gamma^{(4)}\lambda) - \\
&\quad - \frac{1}{640}K_{(1)}(\gamma^{(1)}\mu) - \frac{13}{11520}K_{(3)}(\gamma^{(3)}\mu) - \frac{1}{15360}(\lambda\gamma_{(3)}\lambda)(\gamma^{(3)}\mu) - \frac{1}{1024}\widetilde{T}\,.
\end{aligned} \tag{3.51}$$

Vector-spinors. Contracting with the three independent structures $\delta_a{}^e(\gamma_g)^{\alpha\beta}(\gamma^g)^{\gamma\delta}$, $(\gamma_{ag_1\ldots g_4})^{\alpha\beta}(\gamma^{eg_1\ldots g_4})^{\gamma\delta}$ and $(\gamma_a)^{\alpha\beta}(\gamma^e)^{\gamma\delta}$, we obtain

$$\begin{aligned}
\widetilde{L}^{(2)}_a &= -\frac{9i}{320}(\gamma^{(1)}_a D_{(1)}\mu) - \frac{21}{80}L_{(2)}(\gamma^{(2)}_a\lambda) - \frac{3}{40}L_{(4)}(\gamma^{(4)}_a\lambda) + \\
&\quad + \frac{3}{16}K_{(1)}(\gamma^{(1)}_a\mu) + \frac{63}{320}K_{(3)}(\gamma^{(3)}_a\mu) + \frac{9}{2560}(\lambda\gamma_{(3)}\lambda)(\gamma^{(3)}_a\mu) \\
\widetilde{L}^{(4)}_a &= -\frac{i}{1920}(\gamma^{(1)}_a D_{(1)}\mu) - \frac{1}{480}L_{(2)}(\gamma^{(2)}_a\lambda) + \frac{31}{1680}L_{(4)}(\gamma^{(4)}_a\lambda) - \\
&\quad - \frac{1}{480}K_{(1)}(\gamma^{(1)}_a\mu) + \frac{11}{4480}K_{(3)}(\gamma^{(3)}_a\mu) + \frac{1}{15360}(\lambda\gamma_{(3)}\lambda)(\gamma^{(3)}_a\mu) \\
\widetilde{T}_a &= -\frac{3i}{20}(\gamma^{(1)}_a D_{(1)}\mu) - \frac{1}{5}L_{(2)}(\gamma^{(2)}_a\lambda) + \frac{2}{5}L_{(4)}(\gamma^{(4)}_a\lambda) + \\
&\quad + \frac{1}{5}K_{(1)}(\gamma^{(1)}_a\mu) - \frac{3}{20}K_{(3)}(\gamma^{(3)}_a\mu) + \frac{3}{160}(\lambda\gamma_{(3)}\lambda)(\gamma^{(3)}_a\mu)\,.
\end{aligned} \tag{3.52}$$

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Two-form-spinors. Contracting with the three independent structures $\delta_{[\alpha}{}^{e}(\gamma_{b]})^{\alpha\beta}$ $\delta_{\gamma}{}^{\delta}$, $(\gamma_{ab}{}^{eg_1g_2})^{\alpha\beta}(\gamma_{g_1g_2})_{\gamma}{}^{\delta}$, $\delta_{[\alpha}{}^{e}(\gamma_{b]})^{\delta(\alpha}\delta_{\gamma}{}^{\beta)}$ and projecting onto the gamma-traceless part, we obtain

$$\begin{aligned}
\widetilde{L}^{(2)}_{ab} &= \frac{3}{16}\widetilde{T}_{ab} - \frac{1}{24}L_{(2)}(\gamma^{(2)}_{ab}\lambda) - \frac{1}{4}L_{(4)}(\gamma^{(4)}_{ab}\lambda) - \frac{1}{8}K_{(3)}(\gamma^{(3)}_{ab}\mu) \\
\widetilde{L}^{(4)}_{ab} &= \frac{1}{32}\widetilde{T}_{ab} - \frac{1}{432}L_{(2)}(\gamma^{(2)}_{ab}\lambda) - \frac{13}{360}L_{(4)}(\gamma^{(4)}_{ab}\lambda) - \frac{1}{144}K_{(3)}(\gamma^{(3)}_{ab}\mu)\,. \qquad (3.53)
\end{aligned}$$

Three-form-spinors. Contracting with $(\gamma_{abc}{}^{eg})^{\alpha\beta}(\gamma_g)^{\gamma\delta}$ we obtain

$$\widetilde{L}^{(4)}_{abc} = -\frac{1}{168}L_{(4)}(\gamma^{(4)}_{abc}\lambda) - \frac{1}{336}K_{(3)}(\gamma^{(3)}_{abc}\mu)\,. \qquad (3.54)$$

Four-form-spinors. Contracting with the two independent structures $(\gamma_{abcd}{}^{e})^{\alpha\beta}\delta_{\gamma}{}^{\delta}$ and $(\gamma_{abcd}{}^{e})^{\delta(\alpha}\delta_{\gamma}{}^{\beta)}$, we obtain

$$\widetilde{L}^{(4)}_{abcd} = 0\,. \qquad (3.55)$$

- Case 2.1: $(\underline{\alpha},\underline{\beta},\underline{\delta}) = (\alpha,{}^{\beta},\delta)$

In terms of irreducible representations the BI decomposes as

$$\begin{aligned}
&(10000)\otimes(00001)\otimes(00010)^{2\otimes} \sim \\
&\quad \sim 5(00001)\oplus 7(10010)\oplus 5(01001)\oplus 4(00110)\oplus 2(00012)\oplus\cdots\,.
\end{aligned}$$

Hence the BI imposes at most five constraints on the spinors, seven on the vector-spinors, etc. Let us analyze each representation in turn:

Spinors. Contracting with the five independent structures $(\gamma^e)_{\gamma\beta}\delta_{\delta}{}^{\alpha}$, $(\gamma_g)_{\gamma\beta}$ $(\gamma^{eg})_{\delta}{}^{\alpha}$, $(\gamma^{eg_1g_2})_{\gamma\beta}(\gamma_{g_1g_2})_{\delta}{}^{\alpha}$, $(\gamma_{g_1g_2g_3})_{\gamma\beta}(\gamma^{eg_1g_2g_3})_{\delta}{}^{\alpha}$ and $(\gamma^{eg_1\ldots g_4})_{\gamma\beta}(\gamma_{g_1\ldots g_4})_{\delta}{}^{\alpha}$, we obtain

$$\begin{aligned}
\widetilde{L} &= \widetilde{L}' + \frac{5i}{16}\not{D}\mu + 2L\lambda - \frac{1}{2}L'\lambda + \frac{19}{6}L_{(2)}(\gamma^{(2)}\lambda) + \frac{3}{2}L_{(4)}(\gamma^{(4)}\lambda) + \\
&\quad + \frac{9}{4}K_{(1)}(\gamma^{(1)}\mu) - \frac{9}{16}K_{(3)}(\gamma^{(3)}\mu) + \frac{1}{128}(\lambda\gamma_{(3)}\lambda)(\gamma^{(3)}\mu) \\
\widetilde{L}^{(2)} &= \frac{13i}{960}\not{D}\mu - \frac{1}{5}L\lambda - \frac{1}{120}L'\lambda + \frac{47}{360}L_{(2)}(\gamma^{(2)}\lambda) - \frac{1}{120}L_{(4)}(\gamma^{(4)}\lambda) + \\
&\quad + \frac{5}{48}K_{(1)}(\gamma^{(1)}\mu) + \frac{79}{960}K_{(3)}(\gamma^{(3)}\mu) - \frac{7}{7680}(\lambda\gamma_{(3)}\lambda)(\gamma^{(3)}\mu) \\
\widetilde{L}^{(4)} &= -\frac{i}{11520}\not{D}\mu - \frac{1}{240}L\lambda + \frac{1}{1440}L'\lambda - \frac{1}{4320}L_{(2)}(\gamma^{(2)}\lambda) - \frac{31}{10080}L_{(4)}(\gamma^{(4)}\lambda) - \\
&\quad - \frac{1}{2880}K_{(1)}(\gamma^{(1)}\mu) - \frac{11}{11520}K_{(3)}(\gamma^{(3)}\mu) - \frac{1}{18432}(\lambda\gamma_{(3)}\lambda)(\gamma^{(3)}\mu) \\
\widetilde{K}^{(3)} &= -\frac{i}{360}\not{D}\mu - \frac{1}{36}L'\lambda - \frac{1}{60}L_{(2)}(\gamma^{(2)}\lambda) + \frac{1}{180}L_{(4)}(\gamma^{(4)}\lambda) - \\
&\quad - \frac{2}{45}K_{(1)}(\gamma^{(1)}\mu) + \frac{1}{120}K_{(3)}(\gamma^{(3)}\mu) + \frac{1}{2880}(\lambda\gamma_{(3)}\lambda)(\gamma^{(3)}\mu)
\end{aligned}$$

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$$
\begin{aligned}
\widetilde{T} = & -\frac{8i}{45}\not{D}\mu + \frac{8}{15}L\lambda + \frac{16}{45}L'\lambda - \frac{8}{135}L_{(2)}(\gamma^{(2)}\lambda) - \frac{8}{45}L_{(4)}(\gamma^{(4)}\lambda) - \\
& - \frac{56}{45}K_{(1)}(\gamma^{(1)}\mu) - \frac{8}{45}K_{(3)}(\gamma^{(3)}\mu) - \frac{1}{90}(\lambda\gamma_{(3)}\lambda)(\gamma_a^{(3)}\mu)\,.
\end{aligned} \tag{3.56}
$$

Vector-spinors. Contracting the BI with the seven independent structures $(\gamma^{eg_1g_2g_3})^\gamma{}_\beta(\gamma_{ag_1g_2g_3})_\delta{}^\alpha$, $(\gamma_a{}^{eg_1g_2})^\gamma{}_\beta(\gamma_{g_1g_2})_\delta{}^\alpha$, $(\gamma_{ag})^\gamma{}_\beta(\gamma^{eg})_\delta{}^\alpha$, $(\gamma^{eg})^\gamma{}_\beta(\gamma_{ag})_\delta{}^\alpha$, $\delta_a^e\delta^\gamma{}_\beta$ $\delta_\delta{}^\alpha$, $(\gamma_a{}^{eg_1\cdots g_4})^\gamma{}_\beta(\gamma_{g_1\cdots g_4})_\delta{}^\alpha$ and $(\gamma_{ag_1g_2g_3})^\gamma{}_\beta(\gamma^{eg_1g_2g_3})_\delta{}^\alpha$, we obtain

$$
\begin{aligned}
\widetilde{L}_a^{(2)} &= -\frac{9i}{320}(\gamma_a^{(1)}D_{(1)}\mu) - \frac{21}{80}L_{(2)}(\gamma_a^{(2)}\lambda) - \frac{3}{40}L_{(4)}(\gamma_a^{(4)}\lambda) + \\
&\quad + \frac{3}{16}K_{(1)}(\gamma_a^{(1)}\mu) + \frac{63}{320}K_{(3)}(\gamma_a^{(3)}\mu) + \frac{9}{2560}(\lambda\gamma_{(3)}\lambda)(\gamma_a^{(3)}\mu) \\
\widetilde{L}_a^{(4)} &= -\frac{i}{1920}(\gamma_a^{(1)}D_{(1)}\mu) - \frac{1}{480}L_{(2)}(\gamma_a^{(2)}\lambda) + \frac{31}{1680}L_{(4)}(\gamma_a^{(4)}\lambda) - \\
&\quad - \frac{1}{480}K_{(1)}(\gamma_a^{(1)}\mu) + \frac{11}{4480}K_{(3)}(\gamma_a^{(3)}\mu) + \frac{1}{15360}(\lambda\gamma_{(3)}\lambda)(\gamma_a^{(3)}\mu) \\
\widetilde{K}_a^{(3)} &= -\frac{i}{160}(\gamma_a^{(1)}D_{(1)}\mu) - \frac{1}{40}L_{(2)}(\gamma_a^{(2)}\lambda) + \frac{1}{20}L_{(4)}(\gamma_a^{(4)}\lambda) + \\
&\quad + \frac{1}{40}K_{(1)}(\gamma_a^{(1)}\mu) + \frac{81}{1120}K_{(3)}(\gamma_a^{(3)}\mu) + \frac{1}{1280}(\lambda\gamma_{(3)}\lambda)(\gamma_a^{(3)}\mu) \\
\widetilde{T}_a &= -\frac{3i}{20}(\gamma_a^{(1)}D_{(1)}\mu) - \frac{1}{5}L_{(2)}(\gamma_a^{(2)}\lambda) + \frac{2}{5}L_{(4)}(\gamma_a^{(4)}\lambda) + \\
&\quad + \frac{1}{5}K_{(1)}(\gamma_a^{(1)}\mu) - \frac{3}{20}K_{(3)}(\gamma_a^{(3)}\mu) + \frac{3}{160}(\lambda\gamma_{(3)}\lambda)(\gamma_a^{(3)}\mu)\,.
\end{aligned} \tag{3.57}
$$

Two-form-spinors. Contracting with the five independent structures $(\gamma^e)_{\gamma\beta}$ $(\gamma_{ab})_\delta{}^\alpha$, $(\gamma_{ab}{}^{eg_1g_2})_{\gamma\beta}(\gamma_{g_1g_2})_\delta{}^\alpha$, $(\gamma_{[a}{}^{eg})_{\gamma\beta}(\gamma_{b]g})_\delta{}^\alpha$, $(\gamma_g)_{\gamma\beta}(\gamma_{ab}{}^{eg})_\delta{}^\alpha$ and $(\gamma^{eg_1g_2})_{\gamma\beta}$ $(\gamma_{abg_1g_2})_\delta{}^\alpha$, we obtain

$$
\begin{aligned}
\widetilde{L}_{ab}^{(2)} &= \frac{3}{16}\widetilde{T}_{ab} - \frac{1}{24}L_{(2)}(\gamma_{ab}^{(2)}\lambda) - \frac{1}{4}L_{(4)}(\gamma_{ab}^{(4)}\lambda) - \frac{1}{8}K_{(3)}(\gamma_{ab}^{(3)}\mu) \\
\widetilde{L}_{ab}^{(4)} &= \frac{1}{32}\widetilde{T}_{ab} - \frac{1}{432}L_{(2)}(\gamma_{ab}^{(2)}\lambda) - \frac{13}{360}L_{(4)}(\gamma_{ab}^{(4)}\lambda) - \frac{1}{144}K_{(3)}(\gamma_{ab}^{(3)}\mu) \\
\widetilde{K}_{ab}^{(3)} &= -\frac{1}{8}\widetilde{T}_{ab} - \frac{1}{108}L_{(2)}(\gamma_{ab}^{(2)}\lambda) + \frac{1}{6}L_{(4)}(\gamma_{ab}^{(4)}\lambda) + \frac{1}{18}K_{(3)}(\gamma_{ab}^{(3)}\mu)\,.
\end{aligned} \tag{3.58}
$$

Three-form-spinors. Contracting with the four independent structures $\delta^\gamma{}_\beta$ $(\gamma_{abc}{}^e)_\delta{}^\alpha$, $(\gamma_{[a}{}^e)^\gamma{}_\beta(\gamma_{bc]})_\delta{}^\alpha$, $(\gamma^{eg})^\gamma{}_\beta(\gamma_{abcg})_\delta{}^\alpha$ and $(\gamma_{[a}{}^{eg_1g_2})^\gamma{}_\beta(\gamma_{bc]g_1g_2})_\delta{}^\alpha$, we obtain

$$
\begin{aligned}
\widetilde{L}_{abc}^{(4)} &= -\frac{1}{168}L_{(4)}(\gamma_{abc}^{(4)}\lambda) - \frac{1}{336}K_{(3)}(\gamma_{abc}^{(3)}\mu) \\
\widetilde{K}_{abc}^{(3)} &= -\frac{1}{84}K_{(3)}(\gamma_{abc}^{(3)}\mu)\,.
\end{aligned} \tag{3.59}
$$

Four-form-spinors. Contracting with the independent structures $(\gamma^e)_{\gamma\beta}(\gamma_{abcd})_\delta{}^\alpha$ and $(\gamma_{[a}{}^{eg})_{\gamma\beta}(\gamma_{bcd]g})_\delta{}^\alpha$, we obtain

$$
\widetilde{L}_{abcd}^{(4)} = 0\,. \tag{3.60}
$$

- Case 2.2: $(\underline{\alpha},\underline{\beta},\underline{\delta}) = (\alpha, {}^{\beta}, {}^{\delta})$.

  This is related to the previous case by parity-inversion. The analysis proceeds in a similar fashion.

  Spinors. Contracting with the five independent structures $(\gamma^e)^{\gamma\alpha}\delta^{\delta}{}_{\beta}$, $(\gamma_g)^{\gamma\alpha}(\gamma^{eg})^{\delta}{}_{\beta}$, $(\gamma^{eg_1g_2})^{\gamma\alpha}(\gamma_{g_1g_2})^{\delta}{}_{\beta}$,
  $(\gamma_{g_1g_2g_3})^{\gamma\alpha}(\gamma^{eg_1g_2g_3})^{\delta}{}_{\beta}$ and $(\gamma^{eg_1\dots g_4})^{\gamma\alpha}(\gamma_{g_1\dots g_4})^{\delta}{}_{\beta}$, we obtain

$$
\begin{aligned}
\widetilde{L} &= \widetilde{L}' - \frac{5i}{16}\not{D}\lambda + \frac{1}{2}L\mu - 2L'\mu - \frac{19}{6}L_{(2)}(\gamma^{(2)}\mu) + \frac{3}{2}L_{(4)}(\gamma^{(4)}\mu) - \\
&\quad - \frac{9}{4}K_{(1)}(\gamma^{(1)}\lambda) - \frac{9}{16}K_{(3)}(\gamma^{(3)}\lambda) - \frac{1}{128}(\mu\gamma_{(3)}\mu)(\gamma^{(3)}\lambda) \\
\widetilde{L}^{(2)} &= \frac{13i}{960}\not{D}\lambda - \frac{1}{120}L\mu - \frac{1}{5}L'\mu + \frac{47}{360}L_{(2)}(\gamma^{(2)}\mu) + \frac{1}{120}L_{(4)}(\gamma^{(4)}\mu) + \\
&\quad + \frac{5}{48}K_{(1)}(\gamma^{(1)}\lambda) - \frac{79}{960}K_{(3)}(\gamma^{(3)}\lambda) - \frac{7}{7680}(\mu\gamma_{(3)}\mu)(\gamma^{(3)}\lambda) \\
\widetilde{L}^{(4)} &= \frac{i}{11520}\not{D}\lambda - \frac{1}{1440}L\mu + \frac{1}{240}L'\mu + \frac{1}{4320}L_{(2)}(\gamma^{(2)}\mu) - \frac{31}{10080}L_{(4)}(\gamma^{(4)}\mu) + \\
&\quad + \frac{1}{2880}K_{(1)}(\gamma^{(1)}\lambda) - \frac{11}{11520}K_{(3)}(\gamma^{(3)}\lambda) + \frac{1}{18432}(\mu\gamma_{(3)}\mu)(\gamma^{(3)}\lambda) \\
\widetilde{K}^{(3)} &= \frac{i}{360}\not{D}\lambda + \frac{1}{36}L\mu + \frac{1}{60}L_{(2)}(\gamma^{(2)}\mu) + \frac{1}{180}L_{(4)}(\gamma^{(4)}\mu) + \\
&\quad + \frac{2}{45}K_{(1)}(\gamma^{(1)}\lambda) + \frac{1}{120}K_{(3)}(\gamma^{(3)}\lambda) - \frac{1}{2880}(\mu\gamma_{(3)}\mu)(\gamma^{(3)}\lambda) \\
\widetilde{T} &= -\frac{8i}{45}\not{D}\lambda + \frac{16}{45}L\mu + \frac{8}{15}L'\mu - \frac{8}{135}L_{(2)}(\gamma^{(2)}\mu) + \frac{8}{45}L_{(4)}(\gamma^{(4)}\mu) - \\
&\quad - \frac{56}{45}K_{(1)}(\gamma^{(1)}\lambda) + \frac{8}{45}K_{(3)}(\gamma^{(3)}\lambda) - \frac{1}{90}(\mu\gamma_{(3)}\mu)(\gamma_a^{(3)}\lambda)\,.
\end{aligned}
\tag{3.61}
$$

  Vector-spinors. Contracting the BI with the seven independent structures $(\gamma^{eg_1g_2g_3})_{\gamma}{}^{\alpha}(\gamma_{ag_1g_2g_3})^{\delta}{}_{\beta}$, $(\gamma_a{}^{eg_1g_2})_{\gamma}{}^{\alpha}(\gamma_{g_1g_2})^{\delta}{}_{\beta}$, $(\gamma_{ag})_{\gamma}{}^{\alpha}(\gamma^{eg})^{\delta}{}_{\beta}$, $(\gamma^{eg})_{\gamma}{}^{\alpha}(\gamma_{ag})^{\delta}{}_{\beta}$, $\delta^e_a\delta_{\gamma}{}^{\alpha}\delta^{\delta}{}_{\beta}$, $(\gamma_a{}^{eg_1\dots g_4})_{\gamma}{}^{\alpha}(\gamma_{g_1\dots g_4})^{\delta}{}_{\beta}$ and $(\gamma_{ag_1g_2g_3})_{\gamma}{}^{\alpha}(\gamma^{eg_1g_2g_3})^{\delta}{}_{\beta}$, we obtain

$$
\begin{aligned}
\widetilde{L}_a^{(2)} &= -\frac{9i}{320}(\gamma_a^{(1)}D_{(1)}\lambda) - \frac{21}{80}L_{(2)}(\gamma_a^{(2)}\mu) + \frac{3}{40}L_{(4)}(\gamma_a^{(4)}\mu) + \\
&\quad + \frac{3}{16}K_{(1)}(\gamma_a^{(1)}\lambda) - \frac{63}{320}K_{(3)}(\gamma_a^{(3)}\lambda) + \frac{9}{2560}(\mu\gamma_{(3)}\mu)(\gamma_a^{(3)}\lambda) \\
\widetilde{L}_a^{(4)} &= \frac{i}{1920}(\gamma_a^{(1)}D_{(1)}\lambda) + \frac{1}{480}L_{(2)}(\gamma_a^{(2)}\mu) + \frac{31}{1680}L_{(4)}(\gamma_a^{(4)}\mu) + \\
&\quad + \frac{1}{480}K_{(1)}(\gamma_a^{(1)}\lambda) + \frac{11}{4480}K_{(3)}(\gamma_a^{(3)}\lambda) - \frac{1}{15360}(\mu\gamma_{(3)}\mu)(\gamma_a^{(3)}\lambda) \\
\widetilde{K}_a^{(3)} &= \frac{i}{160}(\gamma_a^{(1)}D_{(1)}\lambda) + \frac{1}{40}L_{(2)}(\gamma_a^{(2)}\mu) + \frac{1}{20}L_{(4)}(\gamma_a^{(4)}\mu) - \\
&\quad - \frac{1}{40}K_{(1)}(\gamma_a^{(1)}\lambda) + \frac{81}{1120}K_{(3)}(\gamma_a^{(3)}\lambda) - \frac{1}{1280}(\mu\gamma_{(3)}\mu)(\gamma_a^{(3)}\lambda) \\
\widetilde{T}_a &= -\frac{3i}{20}(\gamma_a^{(1)}D_{(1)}\lambda) - \frac{1}{5}L_{(2)}(\gamma_a^{(2)}\mu) - \frac{2}{5}L_{(4)}(\gamma_a^{(4)}\mu) + \\
&\quad + \frac{1}{5}K_{(1)}(\gamma_a^{(1)}\lambda) + \frac{3}{20}K_{(3)}(\gamma_a^{(3)}\lambda) + \frac{3}{160}(\mu\gamma_{(3)}\mu)(\gamma_a^{(3)}\lambda)\,.
\end{aligned}
\tag{3.62}
$$

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Two-form-spinors. Contracting with the five independent structures $(\gamma^e)^{\gamma\alpha}(\gamma_{ab})^{\delta}{}_{\beta}$, $(\gamma_{ab}{}^{eg_1g_2})^{\gamma\alpha}(\gamma_{g_1g_2})^{\delta}{}_{\beta}$, $(\gamma_{[a}{}^{eg})^{\gamma\alpha}(\gamma_{b]g})^{\delta}{}_{\beta}$, $(\gamma_g)^{\gamma\alpha}(\gamma_{ab}{}^{eg})^{\delta}{}_{\beta}$ and $(\gamma^{eg_1g_2})^{\gamma\alpha}(\gamma_{abg_1g_2})^{\delta}{}_{\beta}$, we obtain

$$\begin{aligned}
\widetilde{L}^{(2)}_{ab} &= \frac{3}{16}\widetilde{T}_{ab} - \frac{1}{24}L_{(2)}(\gamma^{(2)}_{ab}\mu) + \frac{1}{4}L_{(4)}(\gamma^{(4)}_{ab}\mu) + \frac{1}{8}K_{(3)}(\gamma^{(3)}_{ab}\lambda) \\
\widetilde{L}^{(4)}_{ab} &= -\frac{1}{32}\widetilde{T}_{ab} + \frac{1}{432}L_{(2)}(\gamma^{(2)}_{ab}\mu) - \frac{13}{360}L_{(4)}(\gamma^{(4)}_{ab}\mu) - \frac{1}{144}K_{(3)}(\gamma^{(3)}_{ab}\lambda) \\
\widetilde{K}^{(3)}_{ab} &= \frac{1}{8}\widetilde{T}_{ab} + \frac{1}{108}L_{(2)}(\gamma^{(2)}_{ab}\mu) + \frac{1}{6}L_{(4)}(\gamma^{(4)}_{ab}\mu) + \frac{1}{18}K_{(3)}(\gamma^{(3)}_{ab}\lambda)\,.
\end{aligned} \tag{3.63}$$

Three-form-spinors. Contracting with the four independent structures $\delta_{\gamma}{}^{\alpha}(\gamma_{abc}{}^{e})^{\delta}{}_{\beta}$, $(\gamma_{[a}{}^{e})_{\gamma}{}^{\alpha}(\gamma_{bc]})^{\delta}{}_{\beta}$, $(\gamma^{eg})_{\gamma}{}^{\alpha}(\gamma_{abcg})^{\delta}{}_{\beta}$ and $(\gamma_{[a}{}^{eg_1g_2})_{\gamma}{}^{\alpha}(\gamma_{bc]g_1g_2})^{\delta}{}_{\beta}$, we obtain

$$\begin{aligned}
\widetilde{L}^{(4)}_{abc} &= -\frac{1}{168}L_{(4)}(\gamma^{(4)}_{abc}\lambda) - \frac{1}{336}K_{(3)}(\gamma^{(3)}_{abc}\lambda) \\
\widetilde{K}^{(3)}_{abc} &= -\frac{1}{84}K_{(3)}(\gamma^{(3)}_{abc}\lambda)\,.
\end{aligned} \tag{3.64}$$

Four-form-spinors. Contracting with the independent structures $(\gamma^e)^{\gamma\alpha}(\gamma_{abcd})^{\delta}{}_{\beta}$ and $(\gamma_{[a}{}^{eg})^{\gamma\alpha}(\gamma_{bcd]g})^{\delta}{}_{\beta}$, we obtain

$$\widetilde{L}^{(4)}_{abcd} = 0\,. \tag{3.65}$$

- Case 3.1: $(\underline{\alpha}, \underline{\beta}, \underline{\delta}) = ({}^{\alpha}, {}^{\beta}, \delta)$.

This is related to case 1.2 by parity-inversion.

Spinors. Contracting with the independent structures $(\gamma^e)_{\alpha\beta}\delta^{\gamma}{}_{\delta}$, $(\gamma_g)_{\alpha\beta}(\gamma^{ge})^{\gamma}{}_{\delta}$ and $(\gamma^{eg_1\dots g_4})_{\alpha\beta}(\gamma_{g_1\dots g_4})^{\gamma}{}_{\delta}$, we obtain

$$\begin{aligned}
\widetilde{L} &= \widetilde{L}' + \frac{i}{16}\not{D}\lambda - \frac{1}{4}L\mu - \frac{25}{8}L'\mu - \frac{73}{24}L_{(2)}(\gamma^{(2)}\mu) + \frac{9}{8}L_{(4)}(\gamma^{(4)}\mu) + \\
&\quad + \frac{3}{8}K_{(1)}(\gamma^{(1)}\lambda) - \frac{15}{16}K_{(3)}(\gamma^{(3)}\lambda) + \frac{1}{64}(\mu\gamma_{(3)}\mu)(\gamma^{(3)}\lambda) + \frac{135}{64}\widetilde{T} \\
\widetilde{L}^{(2)} &= -\frac{3i}{320}\not{D}\lambda + \frac{3}{80}L\mu - \frac{21}{160}L'\mu + \frac{59}{480}L_{(2)}(\gamma^{(2)}\mu) + \frac{1}{32}L_{(4)}(\gamma^{(4)}\mu) - \\
&\quad - \frac{9}{160}K_{(1)}(\gamma^{(1)}\lambda) - \frac{19}{320}K_{(3)}(\gamma^{(3)}\lambda) - \frac{3}{1280}(\mu\gamma_{(3)}\mu)(\gamma^{(3)}\lambda) - \frac{33}{256}\widetilde{T} \\
\widetilde{L}^{(4)} &= \frac{i}{3840}\not{D}\lambda - \frac{1}{960}L\mu + \frac{7}{1920}L'\mu + \frac{1}{3456}L_{(2)}(\gamma^{(2)}\mu) - \frac{131}{40320}L_{(4)}(\gamma^{(4)}\mu) + \\
&\quad + \frac{1}{640}K_{(1)}(\gamma^{(1)}\lambda) - \frac{13}{11520}K_{(3)}(\gamma^{(3)}\lambda) + \frac{1}{15360}(\mu\gamma_{(3)}\mu)(\gamma^{(3)}\lambda) + \frac{1}{1024}\widetilde{T}
\end{aligned} \tag{3.66}$$

Vector-spinors. Contracting with the three independent structures $\delta_a{}^e(\gamma_g)_{\alpha\beta}(\gamma^g)_{\gamma\delta}$, $(\gamma_{ag_1\dots g_4})_{\alpha\beta}(\gamma^{eg_1\dots g_4})_{\gamma\delta}$ and $(\gamma_a)_{\alpha\beta}(\gamma^e)_{\gamma\delta}$, we obtain

$$\begin{aligned}
\widetilde{L}^{(2)}_{a} &= -\frac{9i}{320}(\gamma^{(1)}_a D_{(1)}\lambda) - \frac{21}{80}L_{(2)}(\gamma^{(2)}_a\mu) + \frac{3}{40}L_{(4)}(\gamma^{(4)}_a\mu) + \\
&\quad + \frac{3}{16}K_{(1)}(\gamma^{(1)}_a\lambda) - \frac{63}{320}K_{(3)}(\gamma^{(3)}_a\lambda) + \frac{9}{2560}(\mu\gamma_{(3)}\mu)(\gamma^{(3)}_a\lambda)
\end{aligned}$$

$$
\begin{aligned}
\widetilde{L}^{(4)}_a &= \frac{i}{1920}(\gamma^{(1)}_a D_{(1)}\lambda) + \frac{1}{480}L_{(2)}(\gamma^{(2)}_a\mu) + \frac{31}{1680}L_{(4)}(\gamma^{(4)}_a\mu) + \\
&\quad + \frac{1}{480}K_{(1)}(\gamma^{(1)}_a\lambda) + \frac{11}{4480}K_{(3)}(\gamma^{(3)}_a\lambda) - \frac{1}{15360}(\mu\gamma_{(3)}\mu)(\gamma^{(3)}_a\lambda) \\
\widetilde{T}_a &= -\frac{3i}{20}(\gamma^{(1)}_a D_{(1)}\lambda) - \frac{1}{5}L_{(2)}(\gamma^{(2)}_a\mu) - \frac{2}{5}L_{(4)}(\gamma^{(4)}_a\mu) + \\
&\quad + \frac{1}{5}K_{(1)}(\gamma^{(1)}_a\lambda) + \frac{3}{20}K_{(3)}(\gamma^{(3)}_a\lambda) + \frac{3}{160}(\mu\gamma_{(3)}\mu)(\gamma^{(3)}_a\lambda)\,. \qquad (3.67)
\end{aligned}
$$

Two-form-spinors. Contracting with the three independent structures $\delta_{[\alpha}{}^e(\gamma_{b]})_{\alpha\beta}\ \delta^\gamma{}_\delta$, $(\gamma_{ab}{}^{eg_1g_2})_{\alpha\beta}(\gamma_{g_1g_2})^\gamma{}_\delta$, $\delta_{[\alpha}{}^e(\gamma_{b]})_{\delta(\alpha}\delta^\gamma_{\beta)}$ and projecting onto the gamma-traceless part, we obtain

$$
\begin{aligned}
\widetilde{L}^{(2)}_{ab} &= \frac{3}{16}\widetilde{T}_{ab} - \frac{1}{24}L_{(2)}(\gamma^{(2)}_{ab}\mu) + \frac{1}{4}L_{(4)}(\gamma^{(4)}_{ab}\mu) + \frac{1}{8}K_{(3)}(\gamma^{(3)}_{ab}\lambda) \\
\widetilde{L}^{(4)}_{ab} &= -\frac{1}{32}\widetilde{T}_{ab} + \frac{1}{432}L_{(2)}(\gamma^{(2)}_{ab}\mu) - \frac{13}{360}L_{(4)}(\gamma^{(4)}_{ab}\mu) - \frac{1}{144}K_{(3)}(\gamma^{(3)}_{ab}\lambda)\,. \qquad (3.68)
\end{aligned}
$$

Three-form-spinors. Contracting with $(\gamma_{abc}{}^{eg})_{\alpha\beta}(\gamma_g)_{\gamma\delta}$ we obtain

$$
\widetilde{L}^{(4)}_{abc} = -\frac{1}{168}L_{(4)}(\gamma^{(4)}_{abc}\mu) - \frac{1}{336}K_{(3)}(\gamma^{(3)}_{abc}\lambda)\,. \qquad (3.69)
$$

Four-form-spinors. Contracting with the two independent structures $(\gamma_{abcd}{}^e)_{\alpha\beta}\ \delta^\gamma{}_\delta$ and $(\gamma_{abcd}{}^e)_{\delta(\alpha}\delta^\gamma{}_{\beta)}$, we obtain

$$
\widetilde{L}^{(4)}_{abcd} = 0\,. \qquad (3.70)
$$

- Case 3.2: $(\underline{\alpha},\underline{\beta},\underline{\delta}) = ({}^{\alpha}{}_{,}{}^{\beta}{}_{,}{}^{\delta})$.

This related to case 1.1 by parity inversion.

Spinors. Contracting with the three independent structures $(\gamma^e)_{\alpha\beta}\delta^\delta{}_\gamma$, $(\gamma_g)_{\alpha\beta}$ $(\gamma^{ge})^\delta{}_\gamma$ and $(\gamma^{eg_1\ldots g_4})_{\alpha\beta}(\gamma_{g_1\ldots g_4})^\delta{}_\gamma$, we obtain

$$
\begin{aligned}
\widetilde{T} &= -\frac{14i}{135}\not{D}\mu + \frac{8}{45}L\lambda + \frac{32}{45}L'\lambda + \frac{136}{405}L_{(2)}(\gamma^{(2)}\lambda) - \frac{8}{45}L_{(4)}(\gamma^{(4)}\lambda) - \\
&\quad - \frac{16}{45}K_{(1)}(\gamma^{(1)}\mu) + \frac{2}{45}K_{(3)}(\gamma^{(3)}\mu) - \frac{1}{60}(\lambda\gamma_{(3)}\lambda)(\gamma^{(3)}_a\mu) \\
\widetilde{K}^{(3)} &= \frac{i}{540}\not{D}\mu - \frac{1}{45}L\lambda - \frac{1}{180}L'\lambda + \frac{13}{1620}L_{(2)}(\gamma^{(2)}\lambda) + \frac{1}{180}L_{(4)}(\gamma^{(4)}\lambda) + \\
&\quad + \frac{1}{90}K_{(1)}(\gamma^{(1)}\mu) + \frac{1}{45}K_{(3)}(\gamma^{(3)}\mu)\,. \qquad (3.71)
\end{aligned}
$$

Vector-spinors. Contracting the BI with the four independent structures $\delta_a{}^e(\gamma_g)_{\alpha\beta}$ $(\gamma^g)_{\delta\gamma}$, $\delta_a{}^e(\gamma_{g_1\ldots g_5})_{\alpha\beta}(\gamma^{g_1\ldots g_5})_{\delta\gamma}$, $(\gamma_a)_{\alpha\beta}(\gamma^e)_{\delta\gamma}$, $(\gamma_{ag_1\ldots g_4})_{\alpha\beta}(\gamma^{eg_1\ldots g_4})_{\delta\gamma}$ and projecting onto the gamma-traceless part, we obtain

$$
\begin{aligned}
\widetilde{T}_a &= -\frac{3i}{20}(\gamma^{(1)}_a D_{(1)}\mu) - \frac{1}{5}L_{(2)}(\gamma^{(2)}_a\lambda) + \frac{2}{5}L_{(4)}(\gamma^{(4)}_a\lambda) + \\
&\quad + \frac{1}{5}K_{(1)}(\gamma^{(1)}_a\mu) - \frac{3}{20}K_{(3)}(\gamma^{(3)}_a\mu) + \frac{3}{160}(\lambda\gamma_{(3)}\lambda)(\gamma^{(3)}_a\mu)
\end{aligned}
$$

$$\widetilde{K}^{(3)}_a = -\frac{i}{160}(\gamma^{(1)}_a D_{(1)}\mu) - \frac{1}{40}L_{(2)}(\gamma^{(2)}_a\lambda) + \frac{1}{20}L_{(4)}(\gamma^{(4)}_a\lambda) + \\ + \frac{1}{40}K_{(1)}(\gamma^{(1)}_a\mu) + \frac{81}{1120}K_{(3)}(\gamma^{(3)}_a\mu) + \frac{1}{1280}(\lambda\gamma_{(3)}\lambda)(\gamma^{(3)}_a\mu)\,. \quad (3.72)$$

The notation in the equation above is a shorthand for the projection onto the irreducible (gamma-traceless) vector-spinor part. Our conventions are explained in appendix A.

Two-form-spinors. Contracting with the two independent structures $(\gamma_{[a})_{\alpha\beta}$ $(\gamma_{b]}{}^{e})^{\delta}{}_{\gamma}$, $(\gamma_{ab}{}^{eg_1g_2})_{\alpha\beta}(\gamma_{g_1g_2})^{\delta}{}_{\gamma}$, we obtain

$$\widetilde{K}^{(3)}_{ab} = -\frac{1}{8}\widetilde{T}_{ab} - \frac{1}{108}L_{(2)}(\gamma^{(2)}_{ab}\lambda) + \frac{1}{6}L_{(4)}(\gamma^{(4)}_{ab}\lambda) + \frac{1}{18}K_{(3)}(\gamma^{(3)}_{ab}\mu)\,. \quad (3.73)$$

As in the previous case, the projections onto the gamma-traceless part of the two-form-spinors are explained in appendix A.

Three-form-spinors. Contracting with the two independent structures $\delta_{[a}{}^{e}(\gamma_{bc]g_1g_2g_3})_{\alpha\beta}(\gamma^{g_1g_2g_3})_{\delta\gamma}$ and $(\gamma_{abc}{}^{eg})_{\alpha\beta}(\gamma_g)_{\delta\gamma}$, we obtain

$$\widetilde{K}^{(3)}_{abc} = -\frac{1}{84}K_{(3)}(\gamma^{(3)}_{abc}\mu)\,. \quad (3.74)$$

Four-form-spinors. Contracting with $(\gamma_{abcd}{}^{e})_{\alpha\beta}\delta^{\delta}{}_{\gamma}$, we obtain

$$\widetilde{L}^{(4)}_{abcd} = 0\,. \quad (3.75)$$

The solution of the (highly overdetermined) system of equations above is given in section 4. Note that the spinor equations for $(\widetilde{L}-\widetilde{L}')^{\alpha}$ and $(\widetilde{L}-\widetilde{L}')_{\alpha}$ (equations (4.14), (4.19), respectively) can actually be solved for $(L-L')$, as we now show. Let us parameterize $(L-L')$ as follows

$$L - L' = \frac{3}{2}(\mu\lambda) + me^{2\phi}\,, \quad (3.76)$$

where m is a massive constant and ϕ is some real scalar superfield (possibly a constant) of canonical mass dimension zero. Plugging (3.76) in (4.14), (4.19) and taking into account the action of the spinor derivative on $\lambda_\alpha, \mu^\alpha$, we obtain

$$m(D_\alpha\phi - \lambda_\alpha) = 0 \quad (3.77)$$

and

$$m(D^\alpha\phi - \mu^\alpha) = 0\,. \quad (3.78)$$

These equations have two possible solutions: (a) $m = 0$ or (b) $m \neq 0$ and $D^\alpha\phi = \mu^\alpha$, $D_\alpha\phi = \lambda_\alpha$. We shall see in section 3.4 that case (b) is that of Romans supergravity. Moreover let us distinguish two further subcases: $(a1)$ $m = 0$, $L = -L'$ and $(a2)$ $m = 0$, $L \neq -L'$. As we show in the following, case $(a1)$ is that of massless IIA while $(a2)$ is that of HLW supergravity.

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Finally, the spinor derivatives of K_a are computed by employing the identities

$$\begin{aligned} 2D_{(\alpha}D_{\beta)}\lambda_\gamma &= -T_{\alpha\beta}{}^{E}D_E\lambda_\gamma - R_{\alpha\beta\gamma}{}^{\delta}\lambda_\delta \\ 2D^{(\alpha}D^{\beta)}\mu^\gamma &= -T^{\alpha\beta E}D_E\mu^\gamma + \mu^\delta R^{\alpha\beta}{}_{\delta}{}^{\gamma}\,. \end{aligned} \tag{3.79}$$

The result is given in section 4.

3.4 Dimension-2 BI

Taking into account our gauge-fixing for the supertorsion components, the first BI at dimension two reads

$$R_{[abc]}{}^{d} = 0\,. \tag{3.80}$$

The second BI at dimension two can be cast in the form

$$D_{\underline{\alpha}}\widetilde{T}_{ab}{}^{\underline{\delta}} = R_{ab\underline{\alpha}}{}^{\underline{\delta}} - 2D_{[a}T_{b]\underline{\alpha}}{}^{\underline{\delta}} - T_{ab}{}^{\epsilon}T_{\epsilon\underline{\alpha}}{}^{\underline{\delta}} - 2T_{\underline{\alpha}[a|}{}^{\underline{\epsilon}}T_{\underline{\epsilon}|b]}{}^{\underline{\delta}} - D_{\underline{\alpha}}(\gamma_{[a}\widetilde{T}_{b]})^{\underline{\delta}} - D_{\underline{\alpha}}(\gamma_{ab}\widetilde{T})^{\underline{\delta}}\,. \tag{3.81}$$

This simply determines the spinor derivative of $\widetilde{T}_{ab}$, provided the right-hand-side of the equation above is consistent with the gamma-tracelessness of $\widetilde{T}_{ab}$. In other words, the right-hand-side of (3.81) should be annihilated by $(\gamma_a)^{\underline{\alpha}}{}_{\underline{\delta}}$. This condition turns out to be equivalent to the system of all equations-of-motion and Bianchi identities for the bosonic fields! To see this in detail, let us distinguish the following cases.

- Case 1: $(\underline{\alpha},\underline{\delta}) = (\alpha,\delta)$.

  For simplicity of presentation, here and in the remainder of this section we shall focus on the bosonic terms, ignoring quadratic and quartic fermionic terms. In this case, the condition for the tracelessness of the right-hand-side of (3.81) can be written as

$$0 = \gamma^c\mathcal{A}_{bc} + \gamma^{cde}\mathcal{A}_{bcde} + \gamma^{cdefg}\mathcal{A}_{bcdefg}\,, \tag{3.82}$$

  where the coefficients in the expansion above are given by

$$\begin{aligned} \mathcal{A}_{bc} &= \eta_{bc}\bigg(-\frac{272}{25}L^2 + \frac{72}{25}LL' - \frac{3i}{5}D^iK_i - \frac{36}{5}K^iK_i - \frac{304}{45}L_{ij}L^{ij} - \\ &\qquad - \frac{192}{5}K_{ijk}K^{ijk} - \frac{576}{5}L_{ijkl}L^{ijkl}\bigg) + \\ &\quad + \frac{27i}{5}D_bK_c + \frac{3i}{5}D_cK_b - 8K_bK_c - \frac{16}{25}LL_{bc} - \frac{304}{25}L'L_{bc} - \frac{64}{9}L_b{}^iL_{ci} + \\ &\quad + \frac{18i}{5}D^iK_{ibc} + \frac{144}{5}K^iK_{ibc} + 72K_b{}^{ij}K_{cij} + \frac{384}{5}L^{ij}L_{ijbc} - 1536L_b{}^{ijk}L_{cijk} + \\ &\quad + \frac{24}{5}\varepsilon_{bci_1\ldots i_8}L^{i_1\ldots i_4}L^{i_5\ldots i_8} - \frac{1}{2}R_{bc} \\ \mathcal{A}_{bcde} &= \eta_{b[c}\bigg(\frac{3i}{5}D_dK_{e]} - \frac{24}{5}K^iK_{i|de]} - \frac{192}{5}K^iK_{i|de]} - \frac{92}{25}LL_{|de]} + \frac{52}{25}L'L_{|de]} - \\ &\qquad - \frac{512}{5}L^{ij}L_{ij|de]} - \frac{32}{5}\varepsilon_{|de]i_1\ldots i_8}L^{i_1\ldots i_4}L^{i_5\ldots i_8}\bigg) - \frac{1}{4}R_{b[cde]} - \\ &\quad - \frac{12i}{5}D_{[b}K_{cde]} - \frac{48}{5}K_{[b}K_{cde]} + \frac{1488}{25}LL_{bcde} - \frac{1008}{25}L'L_{bcde} \end{aligned}$$

$$\mathcal{A}_{bcdefg} = \eta_{b[c}\Big(\frac{8i}{5}D_d K_{efg]} + \frac{32}{5}K_{|d}K_{efg]} - \frac{272}{25}LL_{|defg]} - \frac{48}{25}L'L_{|defg]}\Big)\,. \tag{3.83}$$

To arrive at these expressions we have Hodge-dualized the $\gamma^{(7)}$ and $\gamma^{(9)}$ terms. We have also made use of the identity

$$\varepsilon_{bci_1\dots i_8}L^{i_1\dots i_4}L^{i_5\dots i_8}(\gamma_a{}^{bc})_{\alpha\beta} = 16(\gamma^{i_1\dots i_7})_{\alpha\beta}L_{ai_1i_2i_3}L_{i_4\dots i_7}\,. \tag{3.84}$$

- Case 2: $(\underline{\alpha},\underline{\delta}) = (\alpha, {}^{\delta})$.

In this case the tracelessness condition takes the form

$$0 = \mathcal{B}_b + \gamma^{cd}\mathcal{B}_{bcd} + \gamma^{cdef}\mathcal{B}_{bcdef}\,, \tag{3.85}$$

where

$$\begin{aligned}
\mathcal{B}_b &= \frac{27i}{5}D_b L - \frac{308}{25}LK_b - \frac{272}{25}L'K_b + \frac{58i}{15}D^iL_{ib} + \frac{232}{5}K^iL_{ib} + \frac{2784}{5}K^{ijk}L_{ijkb}\\
\mathcal{B}_{bcd} &= \eta_{b[c}\Big(\frac{3i}{5}D_{d]}L + \frac{108}{25}LK_{|d]} + \frac{272}{25}L'K_{|d]} - \frac{38i}{15}D^iL_{i|d]} -\\
&\qquad - \frac{152}{5}K^iL_{i|d]} - \frac{1824}{5}K^{ijk}L_{ijk|d]}\Big) + \frac{36}{25}LK_{cd} - \frac{816}{25}L'K_{cd} -\\
&\quad -\frac{29i}{5}D_{[b}L_{cd]} + \frac{156}{5}D^iL_{ibcd} + \frac{1248}{5}K^iL_{ibcd} - \frac{26}{5}\varepsilon_{bcdi_1\dots i_7}K^{i_1i_2i_3}L^{i_4\dots i_7}\\
\mathcal{B}_{bcdef} &= \eta_{b[c}\Big(- \frac{212}{25}LK_{def} + \frac{272}{25}L'K_{def} + \frac{19i}{5}D_{|d}L_{ef]} -\\
&\qquad - \frac{12i}{5}D^iL_{i|def]} - \frac{96}{5}K^iL_{i|def]} + \frac{2}{5}\varepsilon_{|def]i_1\dots i_7}K^{i_1i_2i_3}L^{i_4\dots i_7}\Big) +\\
&\quad + \frac{52}{3}\Big(L_{[bc}K_{def]} - \frac{3}{4}iD_{[b}L_{cdef]} - 3K_{[b}L_{cdef]}\Big) -\\
&\quad - \frac{1}{30}\varepsilon_{bcdefi_1\dots i_5}\Big(L^{i_1i_2}K^{i_3i_4i_5} - \frac{3i}{4}D^{i_1}L^{i_2\dots i_4} - 3K^{i_1}L^{i_2\dots i_4}\Big)\,.
\end{aligned} \tag{3.86}$$

As in the previous case, we have Hodge-dualized the $\gamma^{(6)}$, $\gamma^{(8)}$, $\gamma^{(10)}$ contributions. Also, we have taken into account the identity

$$\frac{1}{24}\varepsilon_{cdei_1\dots i_7}(\gamma_b{}^{cde})_\alpha{}^\delta K^{i_1i_2i_3}L^{i_4\dots i_7} = -(\gamma_{i_1\dots i_6})_\alpha{}^\delta\Big(L_b{}^{i_1i_2i_3}K^{i_4i_5i_6} + \frac{3}{4}K_b{}^{i_1i_2}L^{i_3\dots i_6}\Big)\,. \tag{3.87}$$

- Case 3: $(\underline{\alpha},\underline{\delta}) = ({}^{\alpha}, \delta)$.

This is similar to the previous case. The gamma-tracelessness condition is of the form

$$0 = \mathcal{C}_b + \gamma^{cd}\mathcal{C}_{bcd} + \gamma^{cdef}\mathcal{C}_{bcdef}\,. \tag{3.88}$$

The coefficients $\mathcal{C}_b$, $\mathcal{C}_{bcd}$ and $\mathcal{C}_{bcdef}$ can be obtained from $\mathcal{B}_b$, $\mathcal{B}_{bcd}$ and $\mathcal{B}_{bcdef}$ respectively, by making the substitutions $L \leftrightarrow L'$, $K_{(3)} \to -K_{(3)}$, $L_4 \to -L_4$ and $\varepsilon_{(10)} \to -\varepsilon_{(10)}$.

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- Case 4: $(\underline{\alpha}, \underline{\delta}) = ({}^{\alpha}{}_{,}{}^{\delta})$.

  This is related to case 1 by parity inversion. The gamma-tracelessness condition takes the form

$$0 = \gamma^{c}\mathcal{D}_{bc} + \gamma^{cde}\mathcal{D}_{bcde} + \gamma^{cdefg}\mathcal{D}_{bcdefg}\,. \tag{3.89}$$

  The coefficients $\mathcal{D}_{bc}$, $\mathcal{D}_{bcde}$ and $\mathcal{D}_{bcdefg}$ above can be obtained from $\mathcal{A}_{bc}$, $\mathcal{A}_{bcde}$ and $\mathcal{A}_{bcdefg}$ respectively, by making the substitutions described in the previous case.

In conclusion, the gamma-tracelessness condition is equivalent to

$$\mathcal{A} = \mathcal{B} = \mathcal{C} = \mathcal{D} = 0\,. \tag{3.90}$$

It is straightforward to recognize that in the case (b) at the end of the previous subsection (i.e. for $L - L' = \frac{3}{2}(\mu\lambda) + me^{2\phi}$, $L + L' = 0$ and $\lambda_{\alpha} = D_{\alpha}\phi$, $\mu^{\alpha} = D^{\alpha}\phi$, equations (3.90) reduce to those of Romans supergravity.

In case $(a1)$ (i.e. for $L - L' = \frac{3}{2}(\mu\lambda)$, $L + L' = 0$) we can see, using the results of the preceding analysis of the BI's, that the super-one-form Λ_{A} defined by

$$\begin{aligned} \Lambda_{\alpha} &:= \lambda_{\alpha} \\ \Lambda^{\alpha} &:= \mu^{\alpha} \\ \Lambda_{a} &:= -2iK_{a}\,, \end{aligned} \tag{3.91}$$

is closed:

$$D\Lambda = 0\,. \tag{3.92}$$

It follows that there exists a scalar superfield ϕ such that $\Lambda = D\phi$. In particular, $\lambda_{\alpha} = D_{\alpha}\phi$, $\mu^{\alpha} = D^{\alpha}\phi$ and $K_{a} = \frac{i}{2}D_{a}\phi$. It is now straightforward to see that in this case equations (3.90) reduce to those of massless type IIA supergravity [23]. Note that (3.92) also holds in the case of Romans supergravity.

In the case where spacetime $\mathcal{M}$ is not simply connected, it may be that $\Lambda = D\phi + \psi$, where ψ is closed but not exact. For example, if $\mathcal{M}$ contains an S^{1} we may take $\psi = mdz$, where z is the S^{1} coordinate and m is a mass parameter. Upon compactification on S^{1}, this would amount to a Scherk-Schwarz reduction. The possibility for this topological modification is the remnant in ten dimensions of the freedom to modify ordinary eleven-dimensional supergravity to MM-theory.

Finally, in case $(a2)$ $(L - L' = \frac{3}{2}(\mu\lambda)$, $L + L' \neq 0)$, equations (3.90) reduce to those of HLW supergravity, summarized in section 4. As we have already remarked at the end of section 3.2, in this case there cannot exist a ϕ such that $D_{\alpha}\phi = \lambda_{\alpha}$, $D^{\alpha}\phi = \mu^{\alpha}$. Note that if we parameterize

$$\begin{aligned} L &= \frac{3}{2}\Big(\Phi + \frac{1}{2}(\mu\lambda)\Big) \\ L' &= \frac{3}{2}\Big(\Phi - \frac{1}{2}(\mu\lambda)\Big)\,, \end{aligned} \tag{3.93}$$

where the massless IIA limit (case (a1) above) is reached at $\Phi \to 0$, equations (3.90) imply in particular that $\Phi =$constant. In equations (4.28, 4.29) of section 4 we have set $\Phi = m$.

3.5 Dimension-$\frac{5}{2}$ BI

It can be seen that the dimension-$\frac{5}{2}$ BI does not introduce any new constraints, other than determining the action of the spinor derivative on the dimension-2 component of the supercurvature. Explicitly:

$$D_{\underline{\alpha}} R_{abcd} = 2D_{[a|} R_{\underline{\alpha}|b]cd} - T_{ab}{}^{\underline{\epsilon}} R_{\underline{\epsilon}\underline{\alpha}cd} + 2T_{[a|\underline{\alpha}}{}^{\underline{\epsilon}} R_{\underline{\epsilon}|b]cd}\,. \tag{3.94}$$

4. Summary

Here we summarize the result of the analysis of the BI's carried out in the preceding section. At each order in canonical mass dimension we give the solution for the components of the torsion and curvature, as well as the action of the spinor derivative on the various superfields at that dimension.[9] At dimension two we have chosen to include the bosonic part of the equations-of-motion and Bianchi identities for the bosonic fields of the relatively less-known HLW supergravity.

Dimension 0.

$$\begin{aligned}
T_{\alpha\beta}{}^{c} &= -i(\gamma^c)_{\alpha\beta}\\
T^{\alpha\beta c} &= -i(\gamma^c)^{\alpha\beta}\\
T_{\alpha}{}^{\beta c} &= 0\,.
\end{aligned} \tag{4.1}$$

Dimension $\frac{1}{2}$.

$$\begin{aligned}
T_{\alpha b}{}^{c} &= T^{\alpha}{}_{b}{}^{c} = 0\\
T_{\alpha\beta}{}^{\gamma} &= -4\Big\{\lambda_{(\alpha}\delta_{\beta)}{}^{\gamma} - \frac{1}{2}(\gamma^e)_{\alpha\beta}(\gamma_e\lambda)^{\gamma}\Big\}\\
T^{\alpha\beta}{}_{\gamma} &= -4\Big\{\mu^{(\alpha}\delta^{\beta)}{}_{\gamma} - \frac{1}{2}(\gamma_e)^{\alpha\beta}(\gamma^e\mu)_{\gamma}\Big\}\\
T_{\alpha\beta\gamma} &= (\gamma^e)_{\alpha\beta}(\gamma_e\mu)_{\gamma}\\
T_{\alpha}{}^{\beta\gamma} &= -\frac{1}{2}\Big\{\delta_{\alpha}{}^{\gamma}\mu^{\beta} - \frac{1}{2}(\gamma^{ef})_{\alpha}{}^{\gamma}(\gamma_{ef}\mu)^{\beta}\Big\}\\
T^{\alpha}{}_{\beta\gamma} &= -\frac{1}{2}\Big\{\delta^{\alpha}{}_{\gamma}\lambda_{\beta} - \frac{1}{2}(\gamma^{ef})^{\alpha}{}_{\gamma}(\gamma_{ef}\lambda)_{\beta}\Big\}\\
T^{\alpha\beta\gamma} &= (\gamma_e)^{\alpha\beta}(\gamma^e\lambda)^{\gamma}\,.
\end{aligned} \tag{4.2}$$

[9] We have checked that our formulæ are compatible with overlapping literature. For example, in order to compare with [14], where the supertorsion components were given up to dimension-$\frac{1}{2}$, one should make the following identifications: $\Lambda_\alpha \to (\mu^\alpha, \lambda_\alpha)$ and

$$(\gamma_{11})_{\alpha}{}^{\beta} \to \begin{pmatrix} -\delta_{\alpha}{}^{\beta} & 0 \\ 0 & \delta^{\alpha}{}_{\beta} \end{pmatrix}.$$

One also needs to take into account the identity

$$(\gamma^{ab})_{(\alpha}{}^{\gamma}(\gamma_{ab}\lambda)_{\beta)} = -10\delta^{\gamma}_{(\alpha}\lambda_{\beta)} + 4\gamma^{a}_{\alpha\beta}(\gamma_a\lambda)^{\gamma}\,.$$

Dimension 1. Torsion:

$$
\begin{aligned}
T_{ab}{}^{c} &= 0 \\
T_{a\alpha}{}^{\beta} &= H^1_{fgh}(\gamma_a{}^{fgh})_\alpha{}^\beta + H^2_{afg}(\gamma^{fg})_\alpha{}^\beta \\
T_a{}^\alpha{}_\beta &= H'^1_{fgh}(\gamma_a{}^{fgh})^\alpha{}_\beta + H'^2_{afg}(\gamma^{fg})^\alpha{}_\beta \\
T_{a\alpha\beta} &= S(\gamma_a)_{\alpha\beta} + F^1_{ef}(\gamma_a{}^{ef})_{\alpha\beta} + F^2_{ae}(\gamma^e)_{\alpha\beta} + G^1_{efgh}(\gamma_a{}^{efgh})_{\alpha\beta} + G^2_{aefg}(\gamma^{efg})_{\alpha\beta} \\
T_a{}^{\alpha\beta} &= -S(\gamma_a)^{\alpha\beta} + F^1_{ef}(\gamma_a{}^{ef})^{\alpha\beta} + F^2_{ae}(\gamma^e)^{\alpha\beta} - G^1_{efgh}(\gamma_a{}^{efgh})^{\alpha\beta} - G^2_{aefg}(\gamma^{efg})^{\alpha\beta}\,.
\end{aligned}
\tag{4.3}
$$

Curvature:

$$
\begin{aligned}
R_{\alpha\beta cd} &= 2iH^1_{efg}(\gamma_{cd}{}^{efg})_{\alpha\beta} + 4iH^2_{cde}(\gamma^e)_{\alpha\beta} \\
R^{\alpha\beta}{}_{cd} &= 2iH'^1_{efg}(\gamma_{cd}{}^{efg})^{\alpha\beta} + 4iH'^2_{cde}(\gamma^e)^{\alpha\beta} \\
R_\alpha{}^\beta{}_{cd} &= -2i\Big\{S(\gamma_{cd})_\alpha{}^\beta + F^1_{ef}(\gamma_{cd}{}^{ef})_\alpha{}^\beta + F^2_{cd}\delta_\alpha{}^\beta + G^1_{efgh}(\gamma_{cd}{}^{efgh})_\alpha{}^\beta + 3G^2_{cdfg}(\gamma^{fg})_\alpha{}^\beta\Big\}\,.
\end{aligned}
\tag{4.4}
$$

Spinor derivatives:

$$
\begin{aligned}
D_\alpha\lambda_\beta &= K_e(\gamma^e)_{\alpha\beta} + K_{efg}(\gamma^{efg})_{\alpha\beta} \\
D_\alpha\mu^\beta &= L\ \delta_\alpha{}^\beta + L_{ef}(\gamma^{ef})_\alpha{}^\beta + L_{efgh}(\gamma^{efgh})_\alpha{}^\beta \\
D^\alpha\lambda_\beta &= L'\ \delta^\alpha{}_\beta + L^{ef}(\gamma_{ef})^\alpha{}_\beta - L^{efgh}(\gamma_{efgh})^\alpha{}_\beta \\
D^\alpha\mu^\beta &= K^e(\gamma_e)^{\alpha\beta} - K^{efg}(\gamma_{efg})^{\alpha\beta}\,,
\end{aligned}
\tag{4.5}
$$

where

$$
\begin{aligned}
S &= -\frac{2i}{5}(L-L') + \frac{3i}{5}(\mu\lambda) \\
F^1_{ab} &= -\frac{2i}{3}L_{ab} \\
F^2_{ab} &= \frac{8i}{3}L_{ab} + i(\mu\gamma_{ab}\lambda) \\
H^1_{abc} &= iK_{abc} \\
H^2_{abc} &= -\frac{i}{8}(\lambda\gamma_{abc}\lambda) - \frac{i}{8}(\mu\gamma_{abc}\mu) \\
H'^1_{abc} &= -iK_{abc} \\
H'^2_{abc} &= -\frac{i}{8}(\lambda\gamma_{abc}\lambda) - \frac{i}{8}(\mu\gamma_{abc}\mu) \\
G^1_{abcd} &= 0 \\
G^2_{abcd} &= 8iL_{abcd}\,.
\end{aligned}
\tag{4.6}
$$

For the scalar fields L, L', we have the following cases[10]

$$
\text{Massless IIA:}\begin{cases} L = \dfrac{3}{4}(\mu\lambda) \\ L' = -\dfrac{3}{4}(\mu\lambda) \end{cases}
$$

[10]In fact, massless IIA is the massless limit of both Romans and HLW supergravity and need not be presented as a separate case.

$$\text{Romans:} \begin{cases} L = \frac{1}{2}me^{2\phi} + \frac{3}{4}(\mu\lambda) \\ L' = -\frac{1}{2}me^{2\phi} - \frac{3}{4}(\mu\lambda) \end{cases}$$
$$\text{HLW:} \begin{cases} L = \frac{3}{2}m + \frac{3}{4}(\mu\lambda) \\ L' = \frac{3}{2}m - \frac{3}{4}(\mu\lambda) \end{cases}, \tag{4.7}$$

where in both Romans and massless IIA supergravities we have (modulo the possibility for the topological modification of massless IIA explained at the end of section 3.4) $D_\alpha\phi = \lambda_\alpha$, $D^\alpha\phi = \mu^\alpha$. In HLW supergravity, $\nexists\ \phi$: $D_\alpha\phi = \lambda_\alpha$, $D^\alpha\phi = \mu^\alpha$.

Dimension $\frac{3}{2}$.

Torsion.

$$T_{ab} = \widetilde{T}_{ab} + \gamma_{[a}\widetilde{T}_{b]} + \gamma_{ab}\widetilde{T}\,, \tag{4.8}$$

where for the right-handed spinors $\widetilde{T}^\alpha$, $\widetilde{T}^\alpha_a$ we have (suppressing spinor indices)

$$\begin{aligned} \widetilde{T} &= \frac{272}{225}L\mu - \frac{8}{25}L'\mu + \frac{8}{9}L_{(2)}(\gamma^{(2)}\mu) + \frac{8}{45}L_{(4)}(\gamma^{(4)}\mu) + \\ &\quad + \frac{8}{9}K_{(1)}(\gamma^{(1)}\lambda) - \frac{16}{45}K_{(3)}(\gamma^{(3)}\lambda) - \frac{11}{450}(\mu\gamma_{(3)}\mu)(\gamma_a^{(3)}\lambda) \\ \widetilde{T}_a &= -\frac{3i}{20}(\gamma_a^{(1)}D_{(1)}\mu) - \frac{1}{5}L_{(2)}(\gamma_a^{(2)}\lambda) + \frac{2}{5}L_{(4)}(\gamma_a^{(4)}\lambda) + \\ &\quad + \frac{1}{5}K_{(1)}(\gamma_a^{(1)}\mu) - \frac{3}{20}K_{(3)}(\gamma_a^{(3)}\mu) + \frac{3}{160}(\lambda\gamma_{(3)}\lambda)(\gamma_a^{(3)}\mu) \end{aligned} \tag{4.9}$$

and similarly for the left-handed spinors $\widetilde{T}_\alpha$, $\widetilde{T}_{a\alpha}$,

$$\begin{aligned} \widetilde{T} &= -\frac{8}{25}L\lambda + \frac{272}{225}L'\lambda + \frac{8}{9}L_{(2)}(\gamma^{(2)}\lambda) - \frac{8}{45}L_{(4)}(\gamma^{(4)}\lambda) + \\ &\quad + \frac{8}{9}K_{(1)}(\gamma^{(1)}\mu) + \frac{16}{45}K_{(3)}(\gamma^{(3)}\mu) - \frac{11}{450}(\lambda\gamma_{(3)}\lambda)(\gamma_a^{(3)}\mu) \\ \widetilde{T}_a &= -\frac{3i}{20}(\gamma_a^{(1)}D_{(1)}\lambda) - \frac{1}{5}L_{(2)}(\gamma_a^{(2)}\mu) - \frac{2}{5}L_{(4)}(\gamma_a^{(4)}\mu) + \\ &\quad + \frac{1}{5}K_{(1)}(\gamma_a^{(1)}\lambda) + \frac{3}{20}K_{(3)}(\gamma_a^{(3)}\lambda) + \frac{3}{160}(\mu\gamma_{(3)}\mu)(\gamma_a^{(3)}\lambda)\,. \end{aligned} \tag{4.10}$$

No confusion should arise from the slight abuse of notation, as it is immediately clear by the right-hand-sides of equations (4.9), (4.10) what the chiralities of the spinors $\widetilde{T}$ are in each case.

Curvature.

$$\begin{aligned} R_{\alpha bcd} &= \frac{i}{2}\big(\gamma_b T_{cd} + \gamma_c T_{bd} - \gamma_d T_{bc}\big)_\alpha \\ R^\alpha{}_{bcd} &= \frac{i}{2}\big(\gamma_b T_{cd} + \gamma_c T_{bd} - \gamma_d T_{bc}\big)^\alpha\,. \end{aligned} \tag{4.11}$$

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Spinor derivatives.

$$
\begin{aligned}
DL &= \widetilde{L} \\
DL' &= \widetilde{L}' \\
DL_{ab} &= \widetilde{L}^{(2)}_{ab\alpha} + \gamma_{[a}\widetilde{L}^{(2)}_{b]} + \gamma_{ab}\widetilde{L}^{(2)} \\
DL_{abcd} &= \widetilde{L}^{(4)}_{abcd\alpha} + \gamma_{[a}\widetilde{L}^{(4)}_{bcd]} + \gamma_{[ab}\widetilde{L}^{(4)}_{cd]} + \gamma_{abc}\widetilde{L}^{(4)}_{d]} + \gamma_{abc}\widetilde{L}^{(4)}\,, \qquad (4.12) \\
DK_a &= \widetilde{K}^{(1)}_{a\alpha} + \gamma_a\widetilde{K}^{(1)} \\
DK_{abc} &= \widetilde{K}^{(3)}_{abc\alpha} + \gamma_{[a}\widetilde{K}^{(3)}_{bc]} + \gamma_{[ab}\widetilde{K}^{(3)}_{c]} + \gamma_{abc}\widetilde{K}^{(3)}\,, \qquad (4.13)
\end{aligned}
$$

where

$$
\begin{aligned}
\widetilde{L} &= \widetilde{L}' + 2L\mu - \frac{7}{2}L'\mu - \frac{3}{2}L_{(2)}(\gamma^{(2)}\mu) + \frac{3}{2}L_{(4)}(\gamma^{(4)}\mu) + \\
&\quad + \frac{3}{2}K_{(1)}(\gamma^{(1)}\lambda) - \frac{3}{2}K_{(3)}(\gamma^{(3)}\lambda) - \frac{1}{32}(\mu\gamma_{(3)}\mu)(\gamma^{(3)}\lambda)\,, \qquad (4.14) \\
\widetilde{L}^{(2)}_{ab} &= \frac{3}{16}\widetilde{T}_{ab} - \frac{1}{24}L_{(2)}(\gamma^{(2)}_{ab}\mu) + \frac{1}{4}L_{(4)}(\gamma^{(4)}_{ab}\mu) + \frac{1}{8}K_{(3)}(\gamma^{(3)}_{ab}\lambda) \\
\widetilde{L}^{(2)}_{a} &= -\frac{9i}{320}(\gamma^{(1)}_a D_{(1)}\mu) - \frac{21}{80}L_{(2)}(\gamma^{(2)}_a\lambda) - \frac{3}{40}L_{(4)}(\gamma^{(4)}_a\lambda) + \\
&\quad + \frac{3}{16}K_{(1)}(\gamma^{(1)}_a\mu) + \frac{63}{320}K_{(3)}(\gamma^{(3)}_a\mu) + \frac{9}{2560}(\lambda\gamma_{(3)}\lambda)(\gamma^{(3)}_a\mu) \\
\widetilde{L}^{(2)} &= -\frac{11}{150}L\mu - \frac{27}{200}L'\mu + \frac{7}{120}L_{(2)}(\gamma^{(2)}\mu) + \frac{1}{120}L_{(4)}(\gamma^{(4)}\mu) - \\
&\quad - \frac{7}{120}K_{(1)}(\gamma^{(1)}\lambda) - \frac{1}{24}K_{(3)}(\gamma^{(3)}\lambda) + \frac{1}{9600}(\mu\gamma_{(3)}\mu)(\gamma^{(3)}\lambda)\,, \qquad (4.15) \\
\widetilde{L}^{(4)}_{abcd} &= 0 \\
\widetilde{L}^{(4)}_{abc} &= -\frac{1}{168}L_{(4)}(\gamma^{(4)}_{abc}\lambda) - \frac{1}{336}K_{(3)}(\gamma^{(3)}_{abc}\mu) \\
\widetilde{L}^{(4)}_{ab} &= -\frac{1}{32}\widetilde{T}_{ab} + \frac{1}{432}L_{(2)}(\gamma^{(2)}_{ab}\mu) - \frac{13}{360}L_{(4)}(\gamma^{(4)}_{ab}\mu) - \frac{1}{144}K_{(3)}(\gamma^{(3)}_{ab}\lambda) \\
\widetilde{L}^{(4)}_{a} &= -\frac{i}{1920}(\gamma^{(1)}_a D_{(1)}\mu) - \frac{1}{480}L_{(2)}(\gamma^{(2)}_a\lambda) + \frac{31}{1680}L_{(4)}(\gamma^{(4)}_a\lambda) - \\
&\quad - \frac{1}{480}K_{(1)}(\gamma^{(1)}_a\mu) + \frac{11}{4480}K_{(3)}(\gamma^{(3)}_a\mu) + \frac{1}{15360}(\lambda\gamma_{(3)}\lambda)(\gamma^{(3)}_a\mu) \\
\widetilde{L}^{(4)} &= -\frac{1}{900}L\mu + \frac{11}{2400}L'\mu - \frac{1}{4320}L_{(2)}(\gamma^{(2)}\mu) - \frac{31}{10080}L_{(4)}(\gamma^{(4)}\mu) - \\
&\quad - \frac{1}{1440}K_{(1)}(\gamma^{(1)}\lambda) - \frac{1}{1440}K_{(3)}(\gamma^{(3)}\lambda) + \frac{7}{115200}(\mu\gamma_{(3)}\mu)(\gamma^{(3)}\lambda) \qquad (4.16)
\end{aligned}
$$

and

$$
\begin{aligned}
\widetilde{K}^{(1)}_a &= \frac{i}{20}(\gamma^{(1)}_a D_{(1)}\mu) + \frac{1}{5}L_{(2)}(\gamma^{(2)}_a\lambda) - \frac{2}{5}L_{(4)}(\gamma^{(4)}_a\lambda) - \\
&\quad - \frac{3}{20}K_{(3)}(\gamma^{(3)}_a\mu) - \frac{1}{160}(\lambda\gamma_{(3)}\lambda)(\gamma^{(3)}_a\mu) \\
\widetilde{K}^{(1)} &= -\frac{1}{25}L\mu + \frac{1}{25}L'\mu - \frac{2}{15}L_{(2)}(\gamma^{(2)}\mu) - \frac{2}{5}L_{(4)}(\gamma^{(4)}\mu) - \\
&\quad - \frac{3}{5}K_{(1)}(\gamma^{(1)}\lambda) - \frac{1}{5}K_{(3)}(\gamma^{(3)}\lambda) + \frac{1}{100}(\mu\gamma_{(3)}\mu)(\gamma^{(3)}\lambda)\,, \qquad (4.17)
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{K}^{(3)}_{abc} &= -\frac{1}{84}K_{(3)}(\gamma^{(3)}_{abc}\mu)\\
\widetilde{K}^{(3)}_{ab} &= \frac{1}{8}\widetilde{T}_{ab} + \frac{1}{108}L_{(2)}(\gamma^{(2)}_{ab}\mu) + \frac{1}{6}L_{(4)}(\gamma^{(4)}_{ab}\mu) + \frac{1}{18}K_{(3)}(\gamma^{(3)}_{ab}\lambda)\\
\widetilde{K}^{(3)}_{a} &= -\frac{i}{160}(\gamma^{(1)}_{a}D_{(1)}\mu) - \frac{1}{40}L_{(2)}(\gamma^{(2)}_{a}\lambda) + \frac{1}{20}L_{(4)}(\gamma^{(4)}_{a}\lambda) +\\
&\quad + \frac{1}{40}K_{(1)}(\gamma^{(1)}_{a}\mu) + \frac{81}{1120}K_{(3)}(\gamma^{(3)}_{a}\mu) + \frac{1}{1280}(\lambda\gamma_{(3)}\lambda)(\gamma^{(3)}_{a}\mu)\\
\widetilde{K}^{(3)} &= \frac{13}{900}L\mu + \frac{1}{75}L'\mu + \frac{1}{540}L_{(2)}(\gamma^{(2)}\mu) + \frac{1}{180}L_{(4)}(\gamma^{(4)}\mu) +\\
&\quad + \frac{1}{90}K_{(1)}(\gamma^{(1)}\lambda) + \frac{1}{60}K_{(3)}(\gamma^{(3)}\lambda) - \frac{1}{7200}(\mu\gamma_{(3)}\mu)(\gamma^{(3)}\lambda)\,, \qquad (4.18)
\end{aligned}
$$

for the right-handed spinors $\widetilde{L}^{\alpha}$, $\widetilde{L}^{(2)\alpha}_{ab}$, etc. Similarly, for the left-handed spinors we have

$$
\begin{aligned}
\widetilde{L} &= \widetilde{L}' + \frac{7}{2}L\lambda - 2L'\lambda + \frac{3}{2}L_{(2)}(\gamma^{(2)}\lambda) + \frac{3}{2}L_{(4)}(\gamma^{(4)}\lambda) -\\
&\quad - \frac{3}{2}K_{(1)}(\gamma^{(1)}\mu) - \frac{3}{2}K_{(3)}(\gamma^{(3)}\mu) + \frac{1}{32}(\lambda\gamma_{(3)}\lambda)(\gamma^{(3)}\mu)\,, \qquad (4.19)\\
\widetilde{L}^{(2)}_{ab} &= \frac{3}{16}\widetilde{T}_{ab} - \frac{1}{24}L_{(2)}(\gamma^{(2)}_{ab}\lambda) - \frac{1}{4}L_{(4)}(\gamma^{(4)}_{ab}\lambda) - \frac{1}{8}K_{(3)}(\gamma^{(3)}_{ab}\mu)\\
\widetilde{L}^{(2)}_{a} &= -\frac{9i}{320}(\gamma^{(1)}_{a}D_{(1)}\lambda) - \frac{21}{80}L_{(2)}(\gamma^{(2)}_{a}\mu) + \frac{3}{40}L_{(4)}(\gamma^{(4)}_{a}\mu) +\\
&\quad + \frac{3}{16}K_{(1)}(\gamma^{(1)}_{a}\lambda) - \frac{63}{320}K_{(3)}(\gamma^{(3)}_{a}\lambda) + \frac{9}{2560}(\mu\gamma_{(3)}\mu)(\gamma^{(3)}_{a}\lambda)\\
\widetilde{L}^{(2)} &= -\frac{27}{200}L\lambda - \frac{11}{150}L'\lambda + \frac{7}{120}L_{(2)}(\gamma^{(2)}\lambda) - \frac{1}{120}L_{(4)}(\gamma^{(4)}\lambda) -\\
&\quad - \frac{7}{120}K_{(1)}(\gamma^{(1)}\mu) + \frac{1}{24}K_{(3)}(\gamma^{(3)}\mu) + \frac{1}{9600}(\lambda\gamma_{(3)}\lambda)(\gamma^{(3)}\mu)\,, \qquad (4.20)\\
\widetilde{L}^{(4)}_{abcd} &= 0\\
\widetilde{L}^{(4)}_{abc} &= -\frac{1}{168}L_{(4)}(\gamma^{(4)}_{abc}\lambda) - \frac{1}{336}K_{(3)}(\gamma^{(3)}_{abc}\lambda)\\
\widetilde{L}^{(4)}_{ab} &= \frac{1}{32}\widetilde{T}_{ab} - \frac{1}{432}L_{(2)}(\gamma^{(2)}_{ab}\lambda) - \frac{13}{360}L_{(4)}(\gamma^{(4)}_{ab}\lambda) - \frac{1}{144}K_{(3)}(\gamma^{(3)}_{ab}\mu)\\
\widetilde{L}^{(4)}_{a} &= \frac{i}{1920}(\gamma^{(1)}_{a}D_{(1)}\lambda) + \frac{1}{480}L_{(2)}(\gamma^{(2)}_{a}\mu) + \frac{31}{1680}L_{(4)}(\gamma^{(4)}_{a}\mu) +\\
&\quad + \frac{1}{480}K_{(1)}(\gamma^{(1)}_{a}\lambda) + \frac{11}{4480}K_{(3)}(\gamma^{(3)}_{a}\lambda) - \frac{1}{15360}(\mu\gamma_{(3)}\mu)(\gamma^{(3)}_{a}\lambda)\\
\widetilde{L}^{(4)} &= -\frac{11}{2400}L\lambda + \frac{1}{900}L'\lambda + \frac{1}{4320}L_{(2)}(\gamma^{(2)}\lambda) - \frac{31}{10080}L_{(4)}(\gamma^{(4)}\lambda) +\\
&\quad + \frac{1}{1440}K_{(1)}(\gamma^{(1)}\mu) - \frac{1}{1440}K_{(3)}(\gamma^{(3)}\mu) - \frac{7}{115200}(\lambda\gamma_{(3)}\lambda)(\gamma^{(3)}\mu) \qquad (4.21)
\end{aligned}
$$

and

$$
\begin{aligned}
\widetilde{K}^{(1)}_{a} &= \frac{i}{20}(\gamma^{(1)}_{a}D_{(1)}\lambda) + \frac{1}{5}L_{(2)}(\gamma^{(2)}_{a}\mu) + \frac{2}{5}L_{(4)}(\gamma^{(4)}_{a}\mu) +\\
&\quad + \frac{3}{20}K_{(3)}(\gamma^{(3)}_{a}\lambda) - \frac{1}{160}(\mu\gamma_{(3)}\mu)(\gamma^{(3)}_{a}\lambda)\\
\widetilde{K}^{(1)} &= \frac{1}{25}L\lambda - \frac{1}{25}L'\lambda - \frac{2}{15}L_{(2)}(\gamma^{(2)}\lambda) + \frac{2}{5}L_{(4)}(\gamma^{(4)}\lambda) -
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{3}{5}K_{(1)}(\gamma^{(1)}\mu)+\frac{1}{5}K_{(3)}(\gamma^{(3)}\mu)+\frac{1}{100}(\lambda\gamma_{(3)}\lambda)(\gamma^{(3)}\mu)\,, \qquad (4.22)\\
\widetilde{K}^{(3)}_{abc} &= -\frac{1}{84}K_{(3)}(\gamma^{(3)}_{abc}\lambda)\\
\widetilde{K}^{(3)}_{ab} &= -\frac{1}{8}\widetilde{T}_{ab}-\frac{1}{108}L_{(2)}(\gamma^{(2)}_{ab}\lambda)+\frac{1}{6}L_{(4)}(\gamma^{(4)}_{ab}\lambda)+\frac{1}{18}K_{(3)}(\gamma^{(3)}_{ab}\mu)\\
\widetilde{K}^{(3)}_{a} &= \frac{i}{160}(\gamma^{(1)}_{a}D_{(1)}\lambda)+\frac{1}{40}L_{(2)}(\gamma^{(2)}_{a}\mu)+\frac{1}{20}L_{(4)}(\gamma^{(4)}_{a}\mu)-\\
&\quad -\frac{1}{40}K_{(1)}(\gamma^{(1)}_{a}\lambda)+\frac{81}{1120}K_{(3)}(\gamma^{(3)}_{a}\lambda)-\frac{1}{1280}(\mu\gamma_{(3)}\mu)(\gamma^{(3)}_{a}\lambda)\\
\widetilde{K}^{(3)} &= -\frac{1}{75}L\lambda-\frac{13}{900}L'\lambda-\frac{1}{540}L_{(2)}(\gamma^{(2)}\lambda)+\frac{1}{180}L_{(4)}(\gamma^{(4)}\lambda)-\\
&\quad -\frac{1}{90}K_{(1)}(\gamma^{(1)}\mu)+\frac{1}{60}K_{(3)}(\gamma^{(3)}\mu)+\frac{1}{7200}(\lambda\gamma_{(3)}\lambda)(\gamma^{(3)}\mu)\,. \qquad (4.23)
\end{aligned}
$$

Fermionic equations-of-motion.

$$\gamma^b T_{ab} = -4\widetilde{T}_a - 9\gamma_a\widetilde{T}\,, \qquad (4.24)$$

where $\widetilde{T}$, $\widetilde{T}_a$ are given in (4.9, 4.10) and

$$
\begin{aligned}
i\not{D}\lambda &= -\frac{24}{5}(L-L')\mu-\frac{16}{3}L_{(2)}(\gamma^{(2)}\mu)-\\
&\quad -12K_{(1)}(\gamma^{(1)}\lambda)+3K_{(3)}(\gamma^{(3)}\lambda)+\frac{3}{40}(\mu\gamma_{(3)}\mu)(\gamma^{(3)}\lambda)\\
i\not{D}\mu &= \frac{24}{5}(L-L')\lambda-\frac{16}{3}L_{(2)}(\gamma^{(2)}\lambda)-\\
&\quad -12K_{(1)}(\gamma^{(1)}\mu)-3K_{(3)}(\gamma^{(3)}\mu)+\frac{3}{40}(\lambda\gamma_{(3)}\lambda)(\gamma^{(3)}\mu)\,. \qquad (4.25)
\end{aligned}
$$

Dimension 2.

Curvature.

$$R_{[abc]d} = R_{a[bcd]} = 0\,. \qquad (4.26)$$

Spinor derivatives.

$$D_{\underline{\alpha}}T_{ab}{}^{\underline{\beta}} = R_{ab\underline{\alpha}}{}^{\underline{\beta}} - 2D_{[a}T_{b]\underline{\alpha}}{}^{\underline{\beta}} - 2T_{\underline{\alpha}[a|}{}^{\underline{\epsilon}}T_{\underline{\epsilon}|b]}{}^{\underline{\beta}} - T_{ab}{}^{\underline{\epsilon}}T_{\underline{\epsilon\alpha}}{}^{\underline{\beta}}\,. \qquad (4.27)$$

Bosonic equations-of-motion (HLW).

$$
\begin{aligned}
R_{ab} &= 18mD_{(a}A_{b)}+36m^2A_aA_b-\frac{1}{2}F_a{}^eF_{be}-144H_a{}^{ef}H_{bef}-\\
&\quad -3072G_a{}^{efg}G_{befg}-\eta_{ab}\Big(36m^2+\frac{1}{4}F_{ef}F^{ef}-48H_{efg}H^{efg}\Big)\\
md*A &= -18m^2A\wedge *A+\frac{1}{4}F\wedge *F-96H\wedge *H+3072G\wedge *G\\
d*F &= -72m^2*A+18mA\wedge *F+4608H\wedge *G\\
d*H &= -12mA\wedge *H+8F\wedge *G+768G\wedge G\\
d*G &= -\frac{3}{2}m*H+12mA\wedge *G+24H\wedge G\,. \qquad (4.28)
\end{aligned}
$$

Bosonic Bianchi identities (HLW).

$$
\begin{aligned}
dA &= F \\
dF &= 0 \\
dH &= 48mG - 6mA \wedge H \\
dG &= 6mA \wedge G - \frac{1}{8} F \wedge H \,,
\end{aligned}
\tag{4.29}
$$

where we have introduced the more conventional notation: $iK_{(1)} = \frac{3}{2}mA$, $L_{(2)} = \frac{3}{16}F$, $iK_{(3)} = H$, $L_{(4)} = G$. This is exactly the gauge-fixed form of the equations presented in [8].[11] HLW can also be obtained by a generalized Scherk-Schwarz reduction of ordinary eleven-dimensional supergravity [12] (see also [24]). We have checked that the equations presented here indeed coincide with those in [12][12].

Dimension $\frac{5}{2}$.

Spinor derivatives

$$
D_{\underline{\alpha}} R_{abcd} = 2D_{[a|} R_{\underline{\alpha}|b]cd} - T_{ab}{}^{\underline{\epsilon}} R_{\underline{\epsilon}\underline{\alpha}cd} + 2T_{[a|\underline{\alpha}}{}^{\underline{\epsilon}} R_{\underline{\epsilon}|b]cd} \,.
\tag{4.30}
$$

5. Conclusions

In this paper we have employed a systematic procedure in order to search for massive deformations of IIA supergravity. It is amusing to think that had we not known about them, we would have been able to discover in this way both Romans and HLW supergravities in one go. The method used here is quite general; it would therefore be of interest to apply it to other supersymmetric systems. It is quite plausible that new massive supergravities can be discovered in this way.

As already mentioned in the introduction, HLW supergravity arises upon compactification of a topologically modified version of eleven-dimensional supergravity, MM-theory. However, it is not known at present whether MM-theory can be given a microscopic quantum-mechanical description. Given that de Sitter space is an (unstable) vacuum of MM-theory, if the latter can somehow be related to M-theory it would provide a mechanism for embedding de Sitter in M-theory. This interesting direction deserves to be pursued further, alongside with more recent proposals for the realization of de Sitter space in string/M-theory [25].

In a step towards this direction, it was shown in [11] that HLW supergravity supports (nonsupersymmetric) multi-zero-brane solutions. It was also argued that these states may

[11]To compare with the equations presented in [8] one has to set the σ field of that reference to zero. Remember that σ is analogous to a Stückelberg field and can be gauged-away for $m \neq 0$. Note also that there is a typographical error in the coefficient of the second term on the right-hand-side of the third equation of [8, (4.3)]: instead of 1/4 it should read 3/4. This was subsequently corrected in [10].

[12]To bring the equations above in the form presented in that reference, one needs to substitute $m \to \frac{1}{2}m$, $F \to F_{(2)}$, $H \to -\frac{1}{24}F_{(3)}$, $G \to \frac{1}{192}F_{(4)}$ and set the pure-gauge field φ of [12] to zero.

indeed be associated with a microscopic description of MM-theory[13] and that the latter should represent an unstable phase of M-theory. A better understanding of the dynamics of these zero-branes is important in testing the proposal of [11]. To that end it would be interesting to construct the world-volume theories of 'massive' kappa-symmetric objects propagating in a HLW background (or perhaps directly in MM-theory)[14], either within the superembedding formalism or by directly imposing kappa symmetry.

A. Gamma-traceless projections

In this appendix we explain our conventions concerning the projections onto the gamma-traceless part of the various form-spinors which appear in the analysis of the BI's at dimension three-half.

Let S be a spinor (it may be either chiral or antichiral) and $\Phi_{(p)}$ be a p-form. The following projections (used in section 3) are gamma-traceless, as the reader may verify:

Vector-spinor.

$$\begin{aligned}
\Phi_{(1)}(\gamma_a^{(1)}S) &:= \Phi_i(\gamma^i{}_a S) + 9\Phi_a S \\
\Phi_{(2)}(\gamma_a^{(2)}S) &:= \Phi_{ij}(\gamma^{ij}{}_a S) + 8\Phi_{ia}(\gamma^i S) \\
\Phi_{(3)}(\gamma_a^{(3)}S) &:= \Phi_{ijk}(\gamma^{ijk}{}_a S) + 7\Phi_{ija}(\gamma^{ij} S) \\
\Phi_{(4)}(\gamma_a^{(4)}S) &:= \Phi_{ijkl}(\gamma^{ijkl}{}_a S) + 6\Phi_{ijka}(\gamma^{ijk} S)\,.
\end{aligned} \tag{A.1}$$

Two-form-spinor.

$$\begin{aligned}
\Phi_{(2)}(\gamma_{ab}^{(2)}S) &:= \Phi_{ij}(\gamma^{ij}{}_{ab} S) + 14\Phi_{i[a}(\gamma^i{}_{b]} S) - 56\Phi_{ab} S \\
\Phi_{(3)}(\gamma_{ab}^{(3)}S) &:= \Phi_{ijk}(\gamma^{ijk}{}_{ab} S) + 12\Phi_{ij[a}(\gamma^{ij}{}_{b]} S) - 42\Phi_{iab}(\gamma^i S) \\
\Phi_{(4)}(\gamma_{ab}^{(4)}S) &:= \Phi_{ijkl}(\gamma^{ijkl}{}_{ab} S) + 10\Phi_{ijk[a}(\gamma^{ijk}{}_{b]} S) - 30\Phi_{ijab}(\gamma^{ij} S)\,.
\end{aligned} \tag{A.2}$$

Three-form-spinor.

$$\begin{aligned}
\Phi_{(3)}(\gamma_{abc}^{(3)}S) &:= \Phi_{ijk}(\gamma^{ijk}{}_{abc} S) + 15\Phi_{ij[a}(\gamma^{ij}{}_{bc]} S) - 90\Phi_{i[ab}(\gamma^i{}_{c]} S) - 210\Phi_{abc} S \\
\Phi_{(4)}(\gamma_{abc}^{(4)}S) &:= \Phi_{ijkl}(\gamma^{ijkl}{}_{abc} S) + 12\Phi_{ijk[a}(\gamma^{ij}{}_{bc]} S) - 60\Phi_{ij[ab}(\gamma^{ij}{}_{c]} S) - 120\Phi_{iabc}(\gamma^i S)
\end{aligned} \tag{A.3}$$

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[13] In [26] it was suggested that a microscopic Matrix-model description of MM-theory may be obtainable by a Euclidean radial reduction, as opposed to the usual dimensional reduction of Matrix theory.

[14] For the case of Romans supergravity, such 'massive' branes were considered in [27, 3, 4].

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